List of Symbols

a — parameter of second-order section
$A(f)$ — attenuation (dB)
$A(\omega)$ — attenuation (dB)
A_p — maximum passband attenuation in specification (dB)
A_s — minimum stopband attenuation in specification (dB)
b — parameter of second-order section
$\mathrm{cd}(u, k)$ — Jacobi elliptic cd function
$\mathrm{cd}^{-1}(v, k)$ — inverse Jacobi elliptic cd function
f — frequency (Hz), or digital frequency (frequency for short), $0 \le f \le \frac{1}{2}$
f_{nQ} — normalized frequency in minimal Q-factor design
f_p — passband edge frequency of designed filter (Hz), or $0 \le f \le \frac{1}{2}$
F_p — passband edge frequency in specification (Hz), or $0 \le f \le \frac{1}{2}$
f_s — stopband edge frequency of designed filter (Hz), or $0 \le f \le \frac{1}{2}$
F_s — stopband edge frequency in specification (Hz), or $0 \le f \le \frac{1}{2}$
$G(f)$ — gain (dB)
$G(\omega)$ — gain (dB)
$h(H(s), t)$ — impulse response
$h_s(H(s), t)$ — step response
$\mathcal{H}(n, \xi, \epsilon, f_p, z)$ — lowpass elliptic transfer function
$\mathcal{H}(n, \xi, \epsilon, p)$ — normalized lowpass transfer function
$\mathcal{H}_{\min Q}(n, \xi, f_p, z)$ — minimal Q-factor lowpass elliptic transfer function
$\mathcal{H}_{\min Q}(n, \xi, p)$ — minimal Q-factor normalized lowpass transfer function
$H(s)$ — transfer function in the s-domain
$H(z)$ — transfer function in the z-domain
i — index $(i = 1, 2, \ldots, n)$
j — the imaginary unit $(j = \sqrt{-1})$
k — modulus of elliptic functions
$K_e(n, \xi, \epsilon, x)$ — elliptic characteristic function
$K_J(k)$ — complete elliptic integral of first kind
K_p — characteristic function passband specification
K_s — characteristic function stopband specification
$L(n, \xi)$ — discrimination factor
$\mathcal{L}^{-1}(H(s))$ — the inverse Laplace transform of $H(s)$
$M(f)$ — magnitude response, $M(f) = |H(e^{j2\pi f})|$
$M(\omega)$ — magnitude response, $M(\omega) = |H(j\omega)|$
n — transfer function order (order for short)
$n_{but}(F_p, F_s, K_p, K_s)$ — minimum Butterworth order
$n_{cheb}(F_p, F_s, K_p, K_s)$ — minimum Chebyshev order
$n_{ellip}(F_p, F_s, K_p, K_s)$ — minimum elliptic order
$n_{\max}$ — maximum order

$n_{\min}$ — minimum order
$n_{minQ}(F_p, F_s, K_p, K_s)$ — minimum order of minimal Q-factor design
p — normalized complex frequency
$q(k)$ — modular constant
$Q(s_i)$ — quality factor of pole/zero s_i
$Q(a, b)$ — quality factor of $H(z) = z^2/(z^2 + bz + a)$
$Q_{\min Q}(n, \xi, i)$ — quality factor of ith pole of minimal Q-factor design
$R(n, \xi, x)$ — elliptic rational function
s — complex frequency, Laplace operator (rad/s)
$\mathrm{sn}(u, k)$ — Jacobi elliptic sn function
$\mathrm{sn}^{-1}(v, k)$ — inverse Jacobi elliptic sn function
$S(n, \xi, \epsilon, i)$ — ith pole of (normalized) lowpass transfer function
S_A — attenuation-limit specification
S_G — gain-limit specification
S_K — characteristic-function-limit specification
S_M — magnitude-limit specification
S_r — magnitude-ripple specification
S_δ — magnitude-tolerance specification
$S_{minQ}(n, \xi, i)$ — ith pole of (normalized) lowpass transfer function for minimal Q-factor design
t — time (s)
x — dimensionless variable
$X(n, \xi, i)$ — ith zero of elliptic rational function
z — complex variable in the z-plane; $z = e^{j2\pi f}$ refers to the unit circle
Z — auxiliary complex variable in the z-plane
$Z_{BL}\left(s, f_p\right)$ — bilinear transformation
δ_1 — passband magnitude ripple
δ_2 — stopband magnitude ripple
δ_p — passband magnitude tolerance
δ_s — stopband magnitude tolerance
$\zeta(n, \xi, \epsilon)$ — auxiliary function
ϵ — ripple factor
ξ — selectivity factor
$\tau_{GD}(H(s), \omega)$ — group delay (s)
$\tau_{GD}(H(z), f)$ — group delay (in samples)
$\Phi(f)$ — phase response, $\Phi(f) = \arg(H(e^{j2\pi f}))$
$\Phi(\omega)$ — phase response, $\Phi(\omega) = \arg(H(j\omega))$
ω — angular frequency (rad/s), $\omega = 2\pi f$
θ — angular digital frequency, $\theta = 2\pi f$
$\lfloor x \rfloor$ — integer, $x \le \lfloor x \rfloor < x + 1$

$\underset{x_1<x<x_2}{\mathrm{FindRoot}}\{\, F(x) = G(x) \,\}$ find real x over interval $x_1 < x < x_2$ by solving $F(x) = G(x)$

$\underset{\substack{x_1<x<x_2 \\ y_1<y<y_2}}{\mathrm{FindRoot}} \left\{ \begin{matrix} F_1(x) = G_1(x) \\ F_2(x) = G_2(x) \end{matrix} \right\}$ find real x over interval $x_1 < x < x_2$ and real y over interval $y_1 < y < y_2$ by solving set of equations $\{F_1(x) = G_1(x),\; F_2(x) = G_2(x)\}$

We append a suffix to designate a quantity related to a specific filter type. For example, we add h to designate highpass filter; thus, A_{ph} is A_p of a highpass filter.

Abbreviations

A/D — analog-to-digital
BIBO — bounded-input bounded-output
BICMOS — Bipolar-CMOS
CC — current conveyors
CCI — first-generation current conveyors
CCII — second-generation current conveyors
CD — compact disc
CMOS — Complementary-metal-oxide-semiconductor
CTLTI — continuous-time linear time-invariant
D1 — Design D1
D2 — Design D2
D3a — Design D3a
D3b — Design D3b
D4a — Design D4a
D4b — Design D4b
D5 — Design D5
DFT — discrete Fourier transform
DTLTI — discrete-time linear time-invariant
DSP — digital signal processing
EMQF — elliptic minimal Q-factor
FIR — finite impulse response
FFT — fast Fourier transform
FLOPS — floating point operations
FPGA — field programmable logic array
GSP — gain-sensitivity product
IDFT — inverse discrete Fourier transform
IIR — infinite impulse response
KCL — Kirchhoff current law
LTI — linear time-invariant
op amp — operational amplifier
OTA — operational transconductance amplifier
PDO — polynomial differential operator
rms — root mean square
ROC — region of convergence
SC — switched capacitor
SNR — signal-to-noise ratio
SQP — sequential quadratic programming
SSP — Signals and systems pack
THD — total harmonic distortion
VLSI — very large scale integration
WDF — wave digital filter