CHAPTER 5

ADVANCED ANALOG FILTER DESIGN CASE STUDIES

This chapter reviews basic definitions of analog filter design. It introduces straightforward procedures to map the filter specification into a design space—that is, a set of ranges for parameters that we use in the filter design. We search this design space for the optimum solution according to given criteria, such as minimal Q-factor.

The principal drawback of the classical analog filter design is in returning only one solution, which can be unacceptable for many practical implementations. We propose an advanced approach to the filter design by using a mixture of symbolic and numeric computation and discrete nonlinear optimization. This approach should provide reduced filter complexity for a desired performance, or better performances for the required complexity.

We conclude this chapter by an application example in which we design a robust selective analog switched-capacitor (SC) filter based on commercially available integrated SC circuits.

5.1 BASIC DEFINITIONS

A *filter* is a system that can be used to modify or reshape the frequency spectrum of a signal according to some prescribed requirements.

An *electrical filter* may be used to amplify or attenuate a range of frequency components (sinusoidal signals) or to reject or isolate one specific frequency component. The applications are numerous: to eliminate signal contamination such as noise in communication systems, to separate relevant from irrelevant frequency components, to bandlimit signals before sampling, to convert sampled signals into continuous-time signal, to improve quality of audio equipment, in time-division to frequency-division multiplex systems, in speech synthesis, in the equalization of transmission lines and cables, in the design of artificial cochleas [16] in audio, video, speech, voiceband modems, control, instrumentation, radio signaling and radar, high definition television, radio modems, seismic modeling, financial modeling, and weather modeling.

Generally, the purpose of most filters is to separate the desired signals from undesired signals or noise. Often, the descriptions of the signals and noise are given in terms of their frequency content or the energy of the signals in the frequency bands. For this reason, the filter specifications are usually given in the frequency domain as magnitude response or by gain or attenuation.

The range of frequencies in which the sinusoidal signals are rejected is called a *stopband*. The range of frequencies in which the sinusoidal signals pass with tolerated distortion is called a *passband*. A region between the passband and stopband, where neither desired nor undesired signals exist or the spectra of the desired and undesired signals are overlapped, can be defined as a *transition region*.

In this chapter we will consider a filter with single passband referred to as *lowpass* filter. All other types of filters (*highpass*, *bandpass*, and *bandstop*) can be easily obtained by simple transformation from the lowpass filter.

Once the filter requirements are known, the filter specification can be established; for example, we specify the passband and stopband edge frequencies and tolerances. Next, we proceed with the analog filter design.

The *design* is a set of processes that starts with the specification and ends with the implementation of an analog (product) filter prototype. It comprises four general steps, as follows:

- Approximation
- Realization
- Study of imperfections
- Implementation

The *approximation* step is the process of generating a transfer function that satisfies the desired specification.

The *realization* step is the process of converting the transfer function of the filter into an electric circuit.

The *study of imperfections* investigates the effects of element imperfections, which determine the highest tolerance that can be tolerated without violating the specification of the filter throughout its working life.

The *implementation* step is constructing the product prototype of the filter in hardware. Decisions to be made involve (a) the type of components and packaging and (b) the methods to be used for the manufacture, testing, and tuning of the filter, and so on.

Usually, those four design steps are considered separately, although they are not independent of each other. The main goal is to find the most economical solution in short time. Which filter is better depends on the hardware used for the implementation. Many different constrains have to be fulfilled. The component tolerances and parasitic effects have significant influence on fulfilling the specification. In this case, classical approaches are not adequate for optimizing both the behavior (performance) and implementation (complexity and cost).

We propose an advanced approach to the analog filter design by using a mixture of symbolic and numeric computation and discrete nonlinear optimization. Opposite to the conventional approaches, which return only one design and hide a wealth of alternative filter designs, the advanced design techniques (that we introduce) find a comprehensive set of optimal designs to represent the infinite solution space.

5.2 SPECIFICATION OF AN ANALOG FILTER

Usually, the filter specification is given, but in many cases the designer has to establish the specification by himself. This is the most important prerequisite for the filter design. Namely, if the specification is too restrictive (e.g., very low passband and stopband tolerances, narrow transition region), the filter may not be feasible.

Special care must be taken in determining the passband and stopband tolerances. For example, if the noise at the output of an amplifier is −60 dB, it will not be reasonable to require 80-dB attenuation in stopband. The generated noise in the filter will be much higher than the attenuated undesired signal.

When down-sampling is performed in a digital filter, the lower or higher half of the spectra has to be rejected in order to prevent aliasing effects. In many cases just 20- or 30-dB attenuation in stopband will be sufficient.

The proper selection of the specification must be done according to the nature of signals (i.e., frequency bands and the corresponding levels of the desired and undesired signals or noise) and the available hardware (element tolerances, parasitic effects, etc.).

In this chapter we assume that the specification is given. Next, we examine the feasibility of the practical filter design. Finally, if the filter is not feasible, we propose the minimum changes in the given specification to make the filter design possible.

In practice, there are several ways in presenting the analog filter specification. Usually, designers of analog filters prefer attenuation or gain expressed in dB, while magnitude tolerances are more convenient for the designers of digital filters.

To provide a unified and consistent design, we adopt one form of presenting the specification.

Let us consider a *linear time-invariant system* (LTI), with the input sine signal $x_{in}(t)$

$$x_{in}(t) = X_m \sin(\omega t + \zeta) \tag{5.1}$$

and the steady-state output signal $y_{out}(t)$,

$$y_{out}(t) = Y_m \sin(\omega t + \eta) \tag{5.2}$$

as shown in Fig. 5.1. The ratio $M(\omega) = Y_m / X_m$ describes the change in the amplitude of the input sine signal; we call $M(\omega)$ the *magnitude response*. With $\Phi(\omega) = \eta - \zeta$ we designate the change in phase of the input sine signal; we call $\Phi(\omega)$ the *phase response*; both quantities are defined at the angular frequency in rad/s, $\omega = 2\pi f$, from the frequency range of interest. Some authors prefer to express $M(\omega)$ and $\Phi(\omega)$ in terms of frequency f in Hz. The *frequency response* of the system is defined as $M(\omega)e^{j\Phi(\omega)}$.

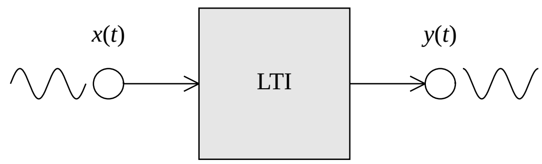

Figure 5.1 Linear time-invariant system.

In other words, $M(\omega)e^{j\Phi(\omega)}$ shows how the input signal is transferred through the system at the specific angular frequency ω rad/s. From the frequency response we can derive the *transfer function*. The transfer function of an LTI system, $H(s)$, is a rational function in the complex variable s; for $s = j\omega$ the transfer function becomes the frequency response $H(j\omega) = M(\omega)e^{j\Phi(\omega)}$.

Several functions are derived from the magnitude response, $M(\omega) = |H(j\omega)|$, and are frequently used in practice. The reciprocal of the squared magnitude is called the *loss function*:

$$L_F(\omega) = \frac{1}{M^2(\omega)} \tag{5.3}$$

We call the function $\sqrt{L_F(\omega) - 1}$ the *characteristic function*:

$$K(\omega) = \sqrt{\frac{1}{M^2(\omega)} - 1} \tag{5.4}$$

Attenuation (in dB) or *loss characteristic* is defined by

$$A(\omega) = 20 \log_{10} \frac{1}{M(\omega)} \tag{5.5}$$

Gain (in dB) is the negative of the attenuation:

$$G(\omega) = -A(\omega) = 20 \log_{10} M(\omega) \tag{5.6}$$

We assume that the maximal value of the magnitude response is 1:

$$\max_{\omega} M(\omega) = \max_{\omega} |H(j\omega)| = 1$$

If this is not the case, the required attenuation or gain can be easily compensated for by multiplying $H(s)$ by a constant.

The magnitude response $M(\omega)$, of an analog lowpass filter, against frequency $f = \frac{1}{2\pi}\omega$, is plotted in Fig. 5.2. Theoretically, upper limit frequency does not exist;

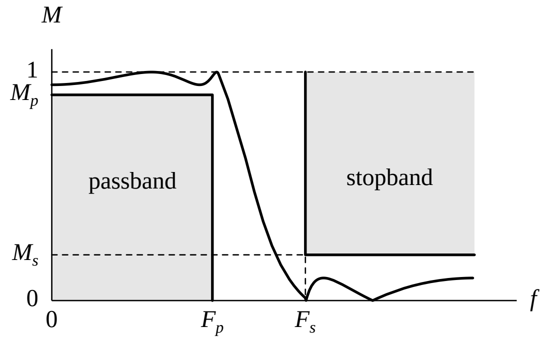

Figure 5.2 Magnitude-limit specification.

however, we can assume that the practical upper frequency is up to 100 times higher than the maximal frequency of interest.

The filter specification can be expressed in several ways:

1. The magnitude limits (Fig. 5.2) define the minimum magnitude in passband, M_p, and the maximum magnitude in stopband, M_s.
2. The magnitude tolerances (Fig. 5.3), specify the maximum magnitude decrease in passband, $\delta_p = 1 - M_p$, and the maximum magnitude in stopband, $\delta_s = M_s$.
3. The magnitude ripple tolerances (Fig. 5.4) describe the maximum magnitude variation, in passband, δ_1, and in stopband, δ_2.
4. The attenuation limits in dB (Fig. 5.5) specify the maximum attenuation in passband, A_p, and the minimum attenuation in stopband, A_s.
5. The gain limits in dB (Fig. 5.6) specify the minimum gain in passband, $G_p = -A_p$, and the maximum gain in stopband, $G_s = -A_s$.

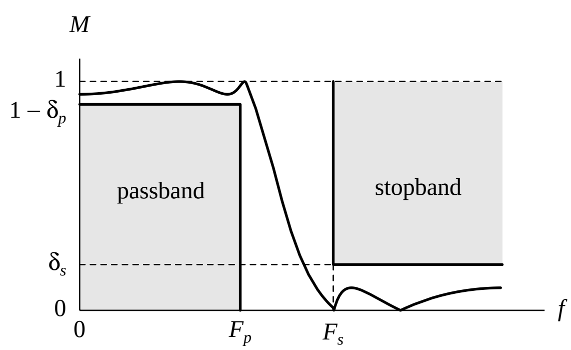

Figure 5.3 Magnitude-tolerance specification.

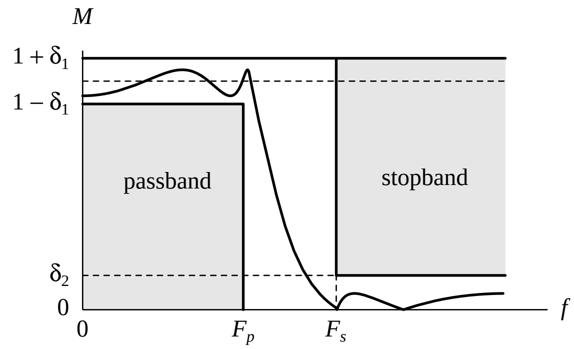

Figure 5.4 Magnitude-ripple specification.

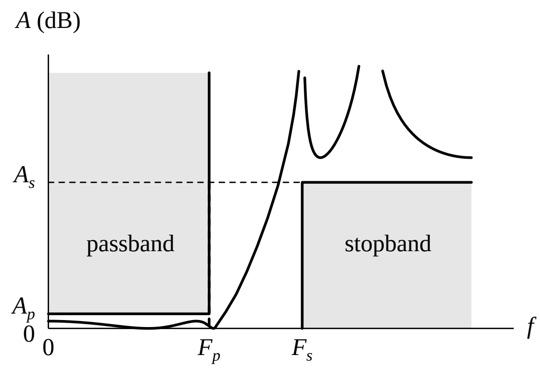

Figure 5.5 Attenuation-limit specification.

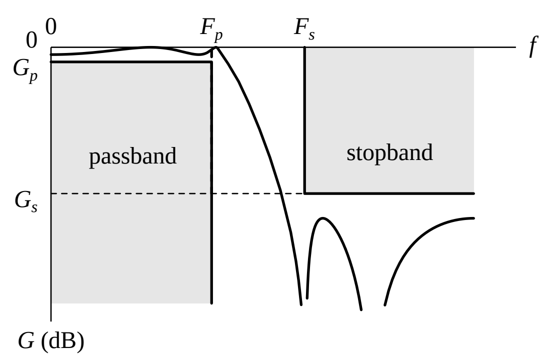

Figure 5.6 Gain-limit specification.

Relations between the specification quantities are summarized in Eqs. (5.7)–(5.12):

$$
\boxed{\begin{array}{ccccccc}
\delta_p & = & 1 - M_p & = & \dfrac{2\delta_1}{1+\delta_1} & = & \dfrac{-1+\sqrt{1+K_p^2}}{\sqrt{1+K_p^2}} \\
1-\delta_p & = & M_p & = & \dfrac{1-\delta_1}{1+\delta_1} & = & \dfrac{1}{\sqrt{1+K_p^2}} \\
\dfrac{\delta_p}{2-\delta_p} & = & \dfrac{1-M_p}{1+M_p} & = & \delta_1 & = & \dfrac{-1+\sqrt{1+K_p^2}}{1+\sqrt{1+K_p^2}} \\
\dfrac{\sqrt{\delta_p(2-\delta_p)}}{1-\delta_p} & = & \dfrac{\sqrt{1-M_p^2}}{M_p} & = & \dfrac{2\sqrt{\delta_1}}{1-\delta_1} & = & K_p \\
-20\log_{10}(1-\delta_p) & = & -20\log_{10} M_p & = & 20\log_{10}\dfrac{1+\delta_1}{1-\delta_1} & = & 10\log_{10}(1+K_p^2) \\
20\log_{10}(1-\delta_p) & = & 20\log_{10} M_p & = & 20\log_{10}\dfrac{1-\delta_1}{1+\delta_1} & = & -10\log_{10}(1+K_p^2)
\end{array}} \tag{5.7}
$$

$$
\boxed{\begin{array}{ccccc}
\dfrac{-1+\sqrt{1+K_p^2}}{\sqrt{1+K_p^2}} & = & 1-10^{-A_p/20} & = & 1-10^{G_p/20} \\
\dfrac{1}{\sqrt{1+K_p^2}} & = & 10^{-A_p/20} & = & 10^{G_p/20} \\
\dfrac{-1+\sqrt{1+K_p^2}}{1+\sqrt{1+K_p^2}} & = & \dfrac{1-10^{-A_p/20}}{1+10^{-A_p/20}} & = & \dfrac{1-10^{G_p/20}}{1+10^{G_p/20}} \\
K_p & = & \dfrac{\sqrt{1-10^{-A_p/10}}}{10^{-A_p/20}} & = & \dfrac{\sqrt{1-10^{G_p/10}}}{10^{G_p/20}} \\
10\log_{10}(1+K_p^2) & = & A_p & = & -G_p \\
-10\log_{10}(1+K_p^2) & = & -A_p & = & G_p
\end{array}} \tag{5.8}
$$

$$
\boxed{
\begin{array}{ccccccc}
\delta_s & = & M_s & = & \dfrac{\delta_2}{1+\delta_1} & = & \dfrac{1}{\sqrt{1+K_s^2}} \\[2ex]
\dfrac{2\delta_s}{2-\delta_p} & = & \dfrac{2M_s}{1+M_p} & = & \delta_2 & = & 2\dfrac{\sqrt{\dfrac{1+K_p^2}{1+K_s^2}}}{1+\sqrt{1+K_p^2}} \\[2ex]
\dfrac{\sqrt{1-\delta_s^2}}{\delta_s} & = & \dfrac{\sqrt{1-M_s^2}}{M_s} & = & \dfrac{\sqrt{(1+\delta_1)^2-\delta_2^2}}{\delta_2} & = & K_s \\[2ex]
-20\log_{10}(\delta_s) & = & -20\log_{10} M_s & = & -20\log_{10}\dfrac{\delta_2}{1+\delta_1} & = & 10\log_{10}(1+K_s^2) \\[2ex]
20\log_{10}(\delta_s) & = & 20\log_{10} M_s & = & 20\log_{10}\dfrac{\delta_2}{1+\delta_1} & = & -10\log_{10}(1+K_s^2)
\end{array}
}
\tag{5.9}
$$

$$
\boxed{
\begin{array}{ccccc}
\dfrac{1}{\sqrt{1+K_s^2}} & = & 10^{-A_s/20} & = & 10^{G_s/20} \\[2ex]
2\dfrac{\sqrt{\dfrac{1+K_p^2}{1+K_s^2}}}{1+\sqrt{1+K_p^2}} & = & 2\dfrac{10^{-A_s/20}}{1+10^{-A_p/20}} & = & 2\dfrac{10^{G_p/20}}{1+10^{G_p/20}} \\[2ex]
K_s & = & \dfrac{\sqrt{1-10^{-A_s/10}}}{10^{-A_s/20}} & = & \dfrac{\sqrt{1-10^{G_s/10}}}{10^{G_s/20}} \\[2ex]
10\log_{10}(1+K_s^2) & = & A_s & = & -G_s \\[2ex]
-10\log_{10}(1+K_s^2) & = & -A_s & = & G_s
\end{array}
}
\tag{5.10}
$$

$$
\boxed{
\begin{array}{llll}
\delta_p = 1-\dfrac{\sqrt{2}}{2} & M_p = \dfrac{\sqrt{2}}{2} & & K_p = 1 \\
\delta_s \approx 0.1 & M_s \approx 0.1 & & K_s = 10 \\
\delta_s \approx 0.01 & M_s \approx 0.01 & & K_s = 10^2 \\
\delta_s \approx 0.0001 & M_s \approx 0.0001 & & K_s = 10^4 \\
\delta_s \approx 0.00001 & M_s \approx 0.00001 & & K_s = 10^5 \\
\delta_p \approx 0.005 & M_p \approx 0.995 & \delta_1 \approx 0.0099 & K_p = 1/10 \\
\delta_p \approx 0.00005 & M_p \approx 0.99995 & \delta_1 \approx 0.00001 & K_p = 1/100
\end{array}
}
\tag{5.11}
$$

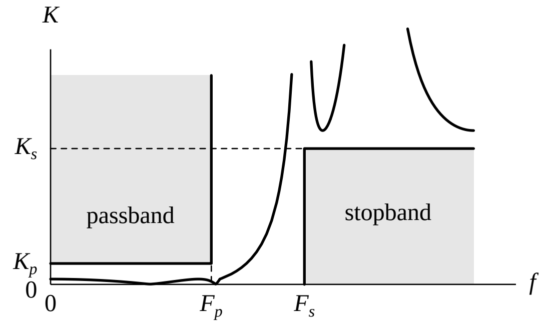

Figure 5.7 Characteristic-function-limit specification.

$$
\begin{array}{lll}
K_p = 1 & A_p \approx 3 & G_p \approx -3 \\
K_s = 10 & A_s \approx 20 & G_s \approx -20 \\
K_s = 10^2 & A_s \approx 40 & G_s \approx -40 \\
K_s = 10^4 & A_s \approx 80 & G_s \approx -80 \\
K_s = 10^5 & A_s \approx 100 & G_s \approx -100 \\
K_p = 1/10 & A_p \approx 0.043 & G_p \approx -0.043 \\
K_p = 1/100 & A_p \approx 0.0004 & G_p \approx -0.0004
\end{array}
\tag{5.12}
$$

The underlining idea of the design that we propose is to map any specification into a new one, expressed in terms of the characteristic function limits (Fig. 5.7) specifying the maximum value in passband, K_p, and the minimum value in stopband, K_s. This provides a unified start for the subsequent design steps.

5.3 APPROXIMATION PROBLEM

In this section we consider the approximation step of the analog filter design.

Let us consider the specification shown in Fig. 5.7. The first step is to generate the characteristic function $K(\omega)$. A function that is a candidate for $K(\omega)$ is known as the *approximation function*, also called the *approximation*. From the approximation the transfer function that meets the specification can be derived. Besides several classical approximations there are numerous closed-form expressions and numerical procedures for generating approximation functions.

The *Butterworth* approximation is smooth and monotonically increases with respect to frequency. It is maximally flat at $\omega = 0$.

The *Chebyshev* approximation, sometimes called the *Chebyshev type I* approximation, gives the smallest ripple over the entire passband. In the stopband, this approximation monotonically increases with respect to frequency.

The Butterworth and Chebyshev type I approximations yield *allpole* transfer functions; they have no transfer function zeros.

The Chebyshev type II approximation, called also the *inverse Chebyshev* approximation, is smooth and monotonically increases with respect to frequency in the passband. It is maximally flat at $\omega = 0$ like the Butterworth approximation. This approximation gives the smallest ripple over the entire stopband.

The *elliptic function approximation*, also called the *elliptic approximation*, the *Cauer approximation* or the *Darlington approximation*, gives the smallest ripple over the entire passband and stopband.

The *Bessel* approximation yields an allpole transfer function like Butterworth and Chebyshev type I. Its magnitude response is smooth and monotonically decreases with respect to frequency. The main characteristics of this approximation are (1) maximally flat group delay at $\omega = 0$ and (2) low step-response overshoot.

Other types of approximations exist, and they exhibit good properties of group delay or time domain [33]. There is a class of approximations that combines the properties of the classical ones; we call this class of approximation functions the *transitional approximations* [49].

Which approximation should the designer choose?

One approach is to analyze all known approximations and to use all known procedures for calculating the magnitude response. The examination of all known approximations has a very high computational cost. Such an approach can be time-consuming, too.

In this chapter we focus on only one approximation, the elliptic approximation. Next, we find the design space—that is, the range of design parameters that satisfy the specification—and keep the design parameters as symbols.

5.4 DESIGN SPACE

In this section we define the design space. First, we map the specification into a standard form. Next, we identify the design parameters. Finally, we calculate the limits of the design parameters.

In the previous section we have shown several ways of presenting required specifications. Any analog lowpass filter can be specified by a set of four quantities as follows:

$$S_\delta = \{F_p, F_s, \delta_p, \delta_s\} \tag{5.13}$$

$$S_M = \{F_p, F_s, M_p, M_s\} \tag{5.14}$$

$$S_r = \{F_p, F_s, \delta_1, \delta_2\} \tag{5.15}$$

$$S_K = \{F_p, F_s, K_p, K_s\} \tag{5.16}$$

$$S_A = \{F_p, F_s, A_p, A_s\} \tag{5.17}$$

$$S_G = \{F_p, F_s, G_p, G_s\} \tag{5.18}$$

and relations between them have been summarized in Eqs. (5.7)–(5.12). The symbol F_p designates the passband edge frequency in Hz, and F_s stands for the stopband edge frequency in Hz.

It is more convenient to transform a given specification S into the specification S_K because it provides a clearer relationship between the design parameters and the specification. Since we have to find a characteristic function $K(\omega)$ in the approximation step, S_K is the most suitable way of presenting the specification.

An infinite number of characteristic functions that fit S exists. We consider the *elliptic approximation*, because it fulfills the requirements with the minimal transfer function order. The minimal order can often lead to the most economical solution, such as the minimal number of components. Also, it will be shown that some other classic approximations are special cases of the elliptic approximation.

The *prototype elliptic approximation*, K_e, is an nth-order rational function in the real variable x:

$$K_e(x) = \epsilon |R(n, \xi, x)| \tag{5.19}$$

where R, referred to as the *elliptic rational function*, satisfies the conditions

$$0 \le |R(n, \xi, x)| \le 1, \qquad |x| \le 1 \tag{5.20}$$

$$L(n, \xi) \le |R(n, \xi, x)|, \qquad |x| \ge \xi \tag{5.21}$$

and L is the *discrimination factor*—that is, the minimal value of the magnitude of R for $|x| \ge \xi$—and can be calculated as

$$L(n, \xi) = |R(n, \xi, \xi)| \tag{5.22}$$

The normalized transition band $1 < x < \xi$ is defined by

$$1 < |R(n, \xi, x)| < L(n, \xi), \qquad 1 < |x| < \xi \tag{5.23}$$

The parameter ξ is called the *selectivity factor*.

The parameter ϵ determines the maximal variation of K_e in the normalized passband $0 \le x \le 1$:

$$0 \le K_e(x) \le \epsilon, \qquad |x| \le 1 \tag{5.24}$$

and is called the *ripple factor*.

The *elliptic approximation*, $K(\omega)$, is a rational function in angular frequency ω rad/s (Figs. 5.8 and 5.9):

$$K(\omega) = K_e(x), \qquad x = \frac{\omega}{2\pi f_p} \tag{5.25}$$

where f_p represents a design parameter that we call the *actual passband edge*. Traditionally, it has been set to $f_p = F_p$.

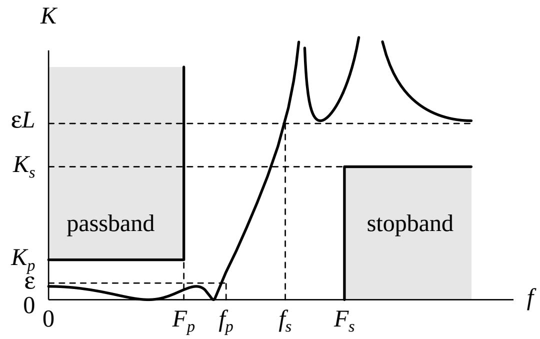

Figure 5.8 Elliptic approximation.

The four quantities n, ξ, ϵ, and f_p are collectively referred to as *design parameters* and can be expressed as a list of the form

$$D = \{n, \xi, \epsilon, f_p\} \tag{5.26}$$

Quantities ξ, ϵ, and f_p take a value over a continuous range of numbers. The order n can take a value from a discrete range of numbers, and is referred to as the *filter order* or *transfer function order*.

It is known [50] that the elliptic approximation provides the minimal order, $n_{min} = n_{ellip}$, for a given specification. The maximal order, from the practical viewpoint, can be assumed to be $n_{max} = 2n_{min}$.

The selectivity factor, ξ, falls within the limits which are found by solving the equations [51]

$$R(n, \xi, \xi) = \frac{K_s}{K_p} \quad \Rightarrow \quad \xi_{min} = \xi_{min}(n) \tag{5.27}$$

$$R\left(n, \xi, \frac{F_s}{F_p}\right) = \frac{K_s}{K_p}, \qquad \xi > \frac{F_s}{F_p} \quad \Rightarrow \quad \xi_{max} = \xi_{max}(n) \tag{5.28}$$

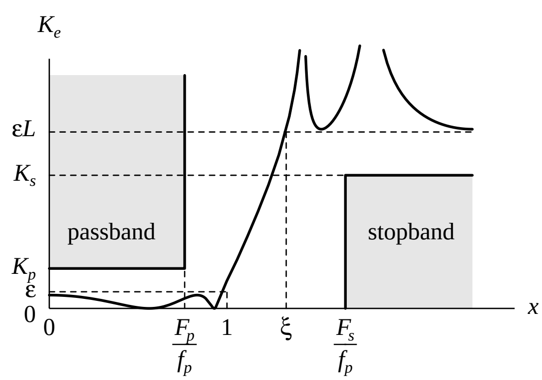

Figure 5.9 Prototype elliptic approximation.

It follows from Eqs. (5.19) and (5.20) that the maximal value of ϵ must be equal to or less than K_p:

$$\epsilon \le K_p \tag{5.29}$$

The ripple factor quantifies the output signal amplitude, Y_m, with respect to the input signal amplitude, X_m. When $K(\omega) = 0$, both amplitudes have the same value; that is,

$$Y_m = X_m \qquad \text{for } K(\omega) = 0 \tag{5.30}$$

With $K(\omega) = \epsilon$ the amplitudes are different:

$$Y_m = \frac{X_m}{\sqrt{1+\epsilon^2}} \qquad \text{for } K(\omega) = \epsilon \tag{5.31}$$

From the previous equations it follows that the smaller the value of ϵ, the smaller the difference between the input and output amplitude.

What is the lower limit of ϵ?

The ripple factor, for $x > \xi$, must meet another condition $K_e(x) \ge K_s$ (Fig. 5.9); thus

$$\epsilon L(n, \xi) \ge K_s \tag{5.32}$$

Therefore, the maximal and minimal values of ϵ has to be determined:

$$\epsilon_{min} \le \epsilon \le \epsilon_{max} \tag{5.33}$$

From Eq. (5.29) we find the upper bound

$$\epsilon_{max} = K_p \tag{5.34}$$

From Eq. (5.32) we determine the lower bound:

$$\epsilon_{min} = \frac{K_s}{L(n,\xi)} = \epsilon_{min}(n, \xi) \tag{5.35}$$

The maximal value of ϵ directly follows from specification, while the minimal value of ϵ depends on the order n and the selectivity factor ξ.

The actual passband edge, f_p, can take a value from the interval

$$f_{p,min} = \frac{F_s}{\xi_{max}} \le f_p \le f_{p,max} = \frac{F_s}{\xi_{min}} \tag{5.36}$$

Obviously, $f_{p,min} = f_{p,min}(n)$, and $f_{p,max} = f_{p,max}(n)$.

The set of all quadruples $D = \{n, \xi, \epsilon, f_p\}$ satisfying the constraints $\{n_{min} \le n \le n_{max}, \xi_{min} \le \xi \le \xi_{max}, \epsilon_{min} \le \epsilon \le \epsilon_{max}, f_{p,min} \le f_p \le f_{p,max}\}$ is called the *design space*:

$$D_S = \{D_{S,n}\}_{n=n_{min}, n_{min}+1, \ldots, n_{max}} \tag{5.37}$$

$$D_{S,n} = \left\{ \begin{array}{ccc} & n & \\ \xi_{min}(n) & \le \xi \le & \xi_{max}(n) \\ \epsilon_{min}(n,\xi) & \le \epsilon \le & K_p \\ f_{p,min}(n) & \le f_p \le & f_{p,max}(n) \end{array} \right\} \tag{5.38}$$

The order n is an integer; it takes only the discrete numeric values, so, it is more convenient to express the design space, D_S, as a list of subspaces, $D_{S,n}$:

$$\left\{\begin{array}{c} \left\{\begin{array}{ccc} & n = n_{min} & \\ \xi_{min}(n) & \leq \xi \leq & \xi_{max}(n) \\ \epsilon_{min}(n,\xi) & \leq \epsilon \leq & K_p \\ f_{p,min}(n) & \leq f_p \leq & f_{p,max}(n) \end{array}\right\} \\ \left\{\begin{array}{ccc} & n = n_{min}+1 & \\ \xi_{min}(n) & \leq \xi \leq & \xi_{max}(n) \\ \epsilon_{min}(n,\xi) & \leq \epsilon \leq & K_p \\ f_{p,min}(n) & \leq f_p \leq & f_{p,max}(n) \end{array}\right\} \\ \cdots \\ \left\{\begin{array}{ccc} & n = n_{max} & \\ \xi_{min}(n) & \leq \xi \leq & \xi_{max}(n) \\ \epsilon_{min}(n,\xi) & \leq \epsilon \leq & K_p \\ f_{p,min}(n) & \leq f_p \leq & f_{p,max}(n) \end{array}\right\} \end{array}\right. \tag{5.39}$$

where

$$\begin{array}{ccccc} 0 & < & \epsilon_{min}(n+1) & < & \epsilon_{min}(n) \\ 1 & < & \xi_{min}(n+1) & < & \xi_{min}(n) \\ \xi_{max}(n) & < & \xi_{max}(n+1) & < & \infty \\ 0 & \leq & f_{p,min}(n+1) & < & f_{p,min}(n) \\ f_{p,max}(n) & < & f_{p,max}(n+1) & < & F_s \end{array} \tag{5.40}$$

5.5 BASIC DESIGN ALTERNATIVES

This section presents our case studies of a comprehensive set of design alternatives based on the design space. It is understood that the rational elliptic function can be readily constructed for a given set of design parameters [52, 53]. The advantages of the various designs are discussed.

Usually, the designer selects the minimal order $n = n_{min}$. The design alternatives that follow are general and valid for any n from the design space. We assume that a specification, S, has been mapped into the form S_K.

5.5.1 Design D1

The design D1 sets the three design parameters, $\xi = F_s/F_p$, $\epsilon = K_p$, $f_p = F_p$, directly from the specification (Fig. 5.10).

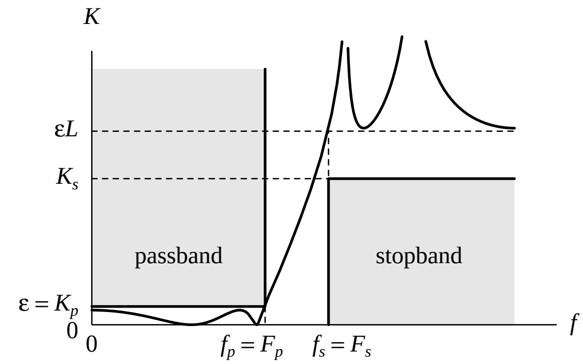

Figure 5.10 Design D1.

This design has higher attenuation in the stopband than is required by the specification. We choose this design when we prefer to achieve as large an attenuation as possible in the stopband—that is, $\epsilon L > K_s$.

5.5.2 Design D2

The design D2 sets the two design parameters, $\xi = F_s/F_p$, $f_p = F_p$, directly from the specification (Fig. 5.11). The ripple factor is computed from $\epsilon = K_s/L(n, \xi)$.

This design has lower attenuation in the passband than is required by the specification. We choose this design when we prefer to achieve as low an attenuation as possible in the passband—that is, $\epsilon < K_p$. Also, this design is suitable when filter element imperfections significantly affect the magnitude response in the passband. In that case, we achieve the highest attenuation margin in the passband (the margin is $K_p - \epsilon$; see Fig. 5.11), and we expect that the imperfections of the implemented filter will not violate the specification.

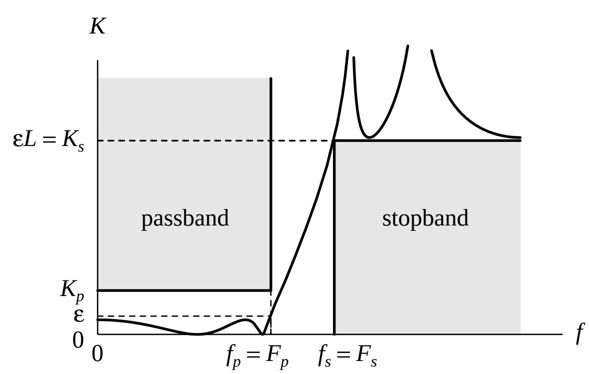

Figure 5.11 Design D2.

5.5.3 Design D3a

In the design D3a we choose the minimal selectivity factor, $\xi = \xi_{min}$, and set the two design parameters, $\epsilon = K_p$, $f_p = F_p$, directly from the specification (Fig. 5.12).

This design has the sharpest magnitude response. When undesired signals exist in the transition region we may prefer design D3a, because it rejects the undesired signals as much as possible.

Disadvantages of D3a can be very high Q-factors and large variation of the group-delay in the passband.

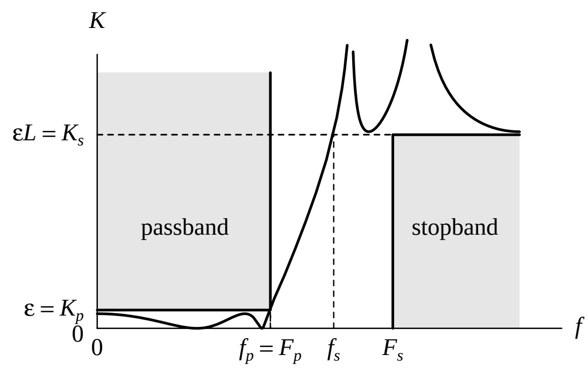

Figure 5.12 Design D3a.

5.5.4 Design D3b

For the design D3b we choose the minimal selectivity factor, $\xi = \xi_{min}$, (the same as in Design 3a) and set the ripple factor, $\epsilon = K_p$, directly from the specification (Fig. 5.13). The actual passband edge is computed from $f_p = f_{p,max} = F_s/\xi$.

This design has the sharpest magnitude response (the same as D3a). When the desired signals exist in the transition region we may prefer the design D3b, because it attenuates the desired signals as low as possible.

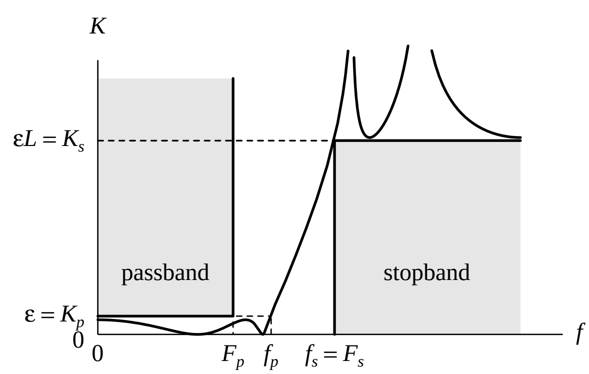

Figure 5.13 Design D3b.

A disadvantage of the design D3b can be very high Q-factors. Although the variation of the group delay can be high, its maximal value can be moved into the transition region, so the group-delay variation can be acceptable in the passband.

5.5.5 Design D4a

In the design D4a we choose the maximal selectivity factor, $\xi = \xi_{max}$, and set the two design parameters, $\epsilon = K_p$ and $f_p = F_p$, directly from the specification (Fig. 5.14).

This design (like the design D1) has higher attenuation in the stopband than is required by the specification, except at the stopband edge frequency. We choose this design when we prefer to achieve as large attenuation as possible in the stopband, i.e. $\epsilon L > K_s$, except at $f = F_s$.

The design D4a has a smoother magnitude response, and that is the main reason for lower Q-factors and smaller variation of the group delay in the passband.

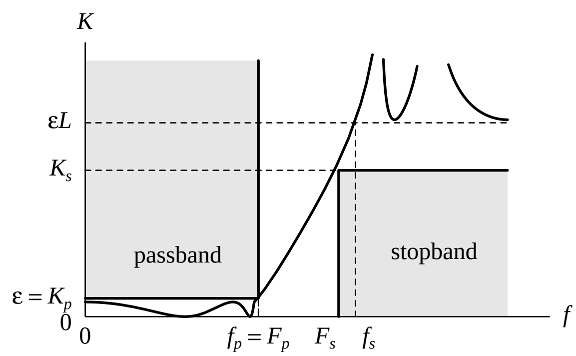

Figure 5.14 Design D4a.

5.5.6 Design D4b

For the design D4b we choose the maximal selectivity factor, $\xi = \xi_{max}$ (the same as in the design D4a) (Fig. 5.15), and calculate the ripple factor from $\epsilon = K_s/L(n, \xi)$. The actual passband edge is computed from $f_p = f_{p,min} = F_s/\xi$.

This design (like the design D2) has lower attenuation in the passband than is required by the specification, except at the passband edge frequency.

We choose this design when we prefer to achieve as low an attenuation as possible in the passband—that is, $\epsilon < K_p$, except at $f = F_p$.

The design D4b has a smoother magnitude response. This design usually yields very low Q-factors and small variation of the group delay in the passband. The design D4b has a very low ripple factor ϵ.

It should be noticed that there exists a straightforward procedure for computing the ripple factor, ϵ, for a given selectivity factor ξ, that yields the minimal Q-factors [47]. We designate this design by D5.

A disadvantage of D3a, D3b, D4a, and D4b is lack of any attenuation margin. Any imperfection, usually in implementation step (such as element tolerances), can violate the specification.

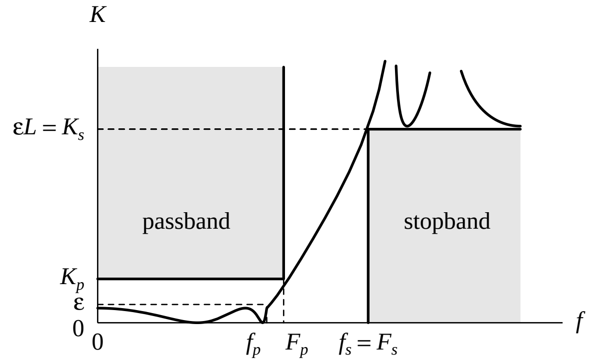

Figure 5.15 Design D4b.

5.5.7 Remarks on Design Alternatives

The approach that we propose in the previous sections we have programmed in *Mathematica* [2]. Several design examples are exercised for an illustrative specification:

$$S_A = \{F_p = 3\text{ kHz}, F_s = 3.225\text{ kHz}, A_p = 0.2\text{ dB}, A_s = 40\text{ dB}\}.$$

First, the attenuation-limit specification $S_A = \{3,\ 3.225,\ 0.2,\ 40\}$ is transformed into the characteristic-function-limit specification

$$S_K = \{3, 3.225, 0.2171, 100\}.$$

Next, all designs are calculated, and the gain is plotted on Figs. 5.16–5.28. The design parameters, the actual stopband edge, the maximal attenuation in the passband, and the minimal attenuation in the stopband, are summarized in Table 5.1 and Table 5.2. The maximal Q-factor is presented in the last column.

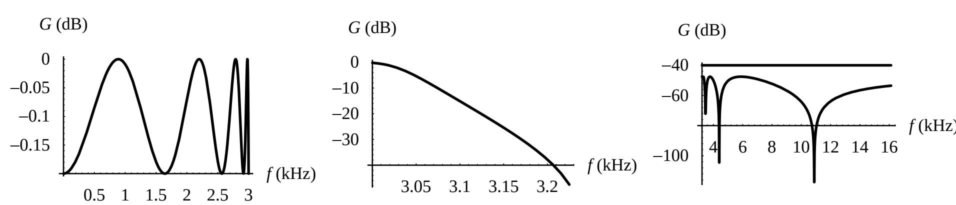

Figure 5.16 Design D1: $D = \{n_{\min}, \frac{F_s}{F_p}, \epsilon_{\max}, F_p\},\ S_A = \{3, 3.225, 0.2, 40\}$.

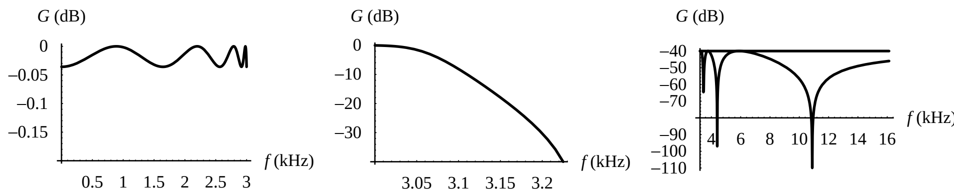

Figure 5.17 Design D2: $D = \{n_{\min}, \frac{F_s}{F_p}, \epsilon < \epsilon_{\max}, F_p\}$, $S_A = \{3, 3.225, 0.2, 40\}$.

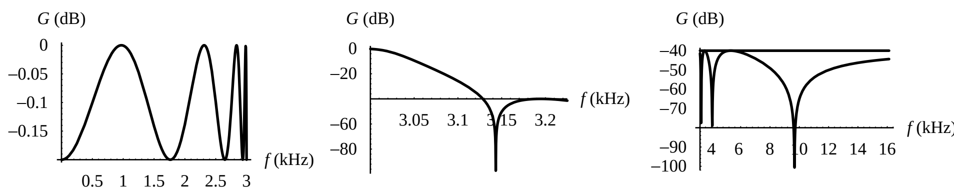

Figure 5.18 Design D3a: $D = \{n_{\min}, \xi_{\min}, \epsilon_{\max}, F_p\}$, $S_A = \{3, 3.225, 0.2, 40\}$.

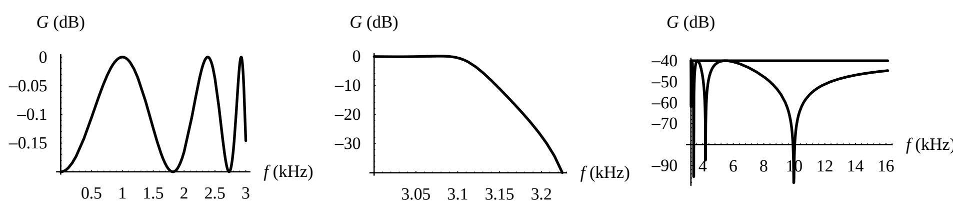

Figure 5.19 Design D3b: $D = \{n_{\min}, \xi_{\min}, \epsilon_{\max}, \frac{F_s}{\xi_{\min}}\}$, $S_A = \{3, 3.225, 0.2, 40\}$.

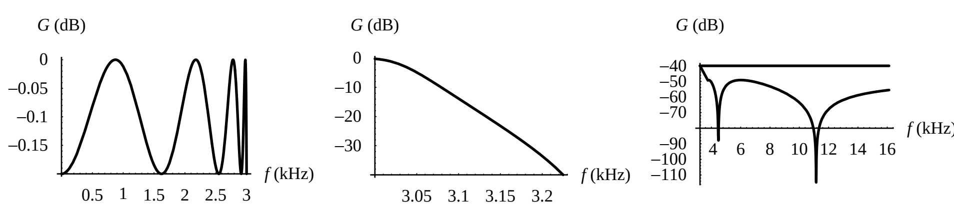

Figure 5.20 Design D4a: $D = \{n_{\min}, \xi_{\max}, \epsilon_{\max}, F_p\}$, $S_A = \{3, 3.225, 0.2, 40\}$.

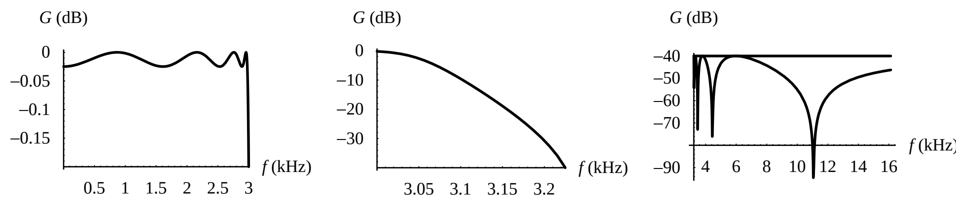

Figure 5.21 Design D4b: $D = \{n_{\min}, \xi_{\max}, \frac{K_s}{L(n_{\min}, \xi_{\max})}, \frac{F_s}{\xi_{\max}}\}$, $S_A = \{3, 3.225, 0.2, 40\}$.

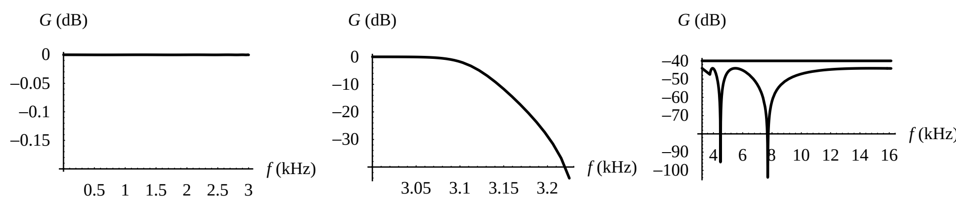

Figure 5.22 Design D5: $D = \{n > n_{\min}, \xi_{\min\,Q}, \epsilon << \epsilon_{\max}, f_p < F_p\}$, $S_A = \{3, 3.225, 0.2, 40\}$.

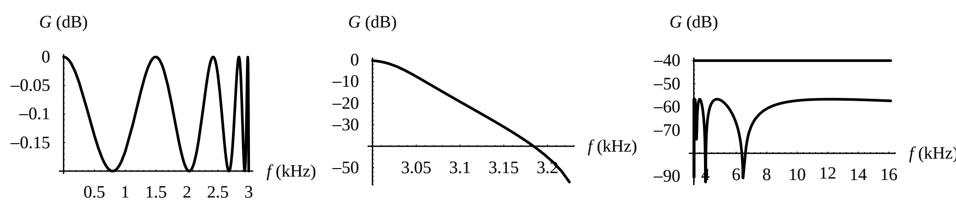

Figure 5.23 Design D1: $D = \{n_{\min} + 1, \frac{F_s}{F_p}, \epsilon_{\max}, F_p\}$, $S_A = \{3, 3.225, 0.2, 40\}$.

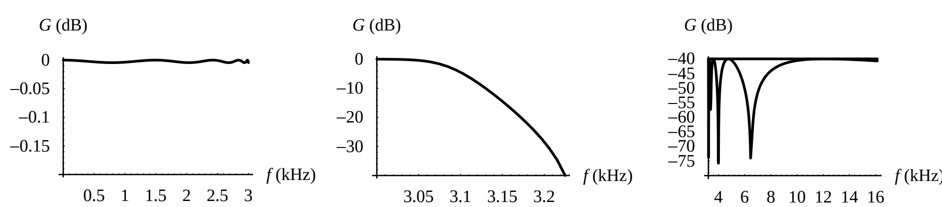

Figure 5.24 Design D2: $D = \{n_{\min} + 1, \frac{F_s}{F_p}, \epsilon < \epsilon_{\max}, F_p\}$, $S_A = \{3, 3.225, 0.2, 40\}$.

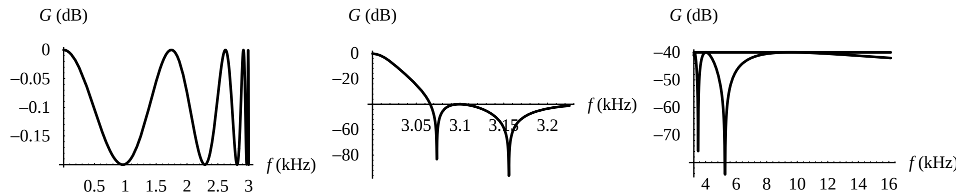

Figure 5.25 Design D3a: $D = \{n_{\min} + 1, \xi_{\min}, \epsilon_{\max}, F_p\},\ S_A = \{3, 3.225, 0.2, 40\}$.

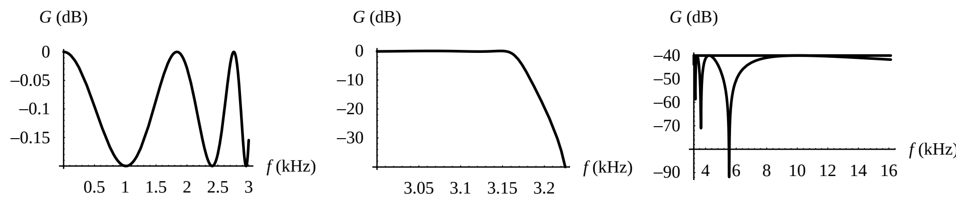

Figure 5.26 Design D3b: $D = \{n_{\min} + 1, \xi_{\min}, \epsilon_{\max}, \frac{F_s}{\xi_{\min}}\}$,
$S_A = \{3, 3.225, 0.2, 40\}$.

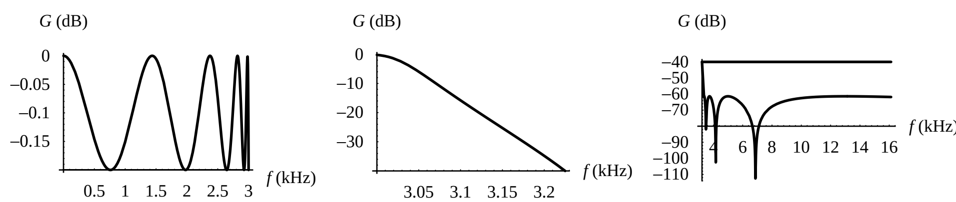

Figure 5.27 Design D4a: $D = \{n_{\min} + 1, \xi_{\max}, \epsilon_{\max}, F_p\},\ S_A = \{3, 3.225, 0.2, 40\}$.

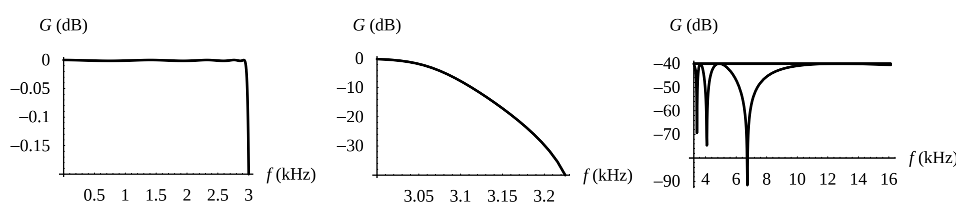

Figure 5.28 Design D4b: $D = \{n_{\min} + 1, \xi_{\max}, \frac{K_s}{L(n, \xi_{\max})}, \frac{F_s}{\xi_{\max}}\}$,
$S_A = \{3, 3.225, 0.2, 40\}$.

Table 5.1 Design summary for $n = n_{min}$

	n	ξ	ϵ	f_p (Hz)	f_s (Hz)	a_p (dB)	a_s (dB)	Q_{max}
D 1	8	1.075	0.2171	3000	3225	0.2	47.55	29.9
D 2	8	1.075	0.09097	3000	3225	0.03579	40.	24.2
D 3a	8	1.043	0.2171	3000	3129	0.2	40.	42.1
D 3b	8	1.043	0.2171	3092	3225	0.2	40.	42.1
D 4a	8	1.083	0.2171	3000	3250	0.2	49.14	28.2
D 4b	8	1.083	0.07579	2977	3225	0.02487	40.	22.1
D 5	10	1.079	0.01	2989	3225	$4.343 \cdot 10^{-4}$	40.	21.3

Table 5.2 Design summary for $n = n_{min} + 1$

	n	ξ	ϵ	f_p (kHz)	f_s (kHz)	a_p (dB)	a_s (dB)	Q_{max}
D 1	9	1.075	0.2171	3.	3.225	0.2	56.66	37.4
D 2	9	1.075	0.03188	3.	3.225	0.004412	40.	25.8
D 3a	9	1.022	0.2171	3.	3.066	0.2	40.	81.2
D 3b	9	1.022	0.2171	3.156	3.225	0.2	40.	81.2
D 4a	9	1.098	0.2171	3.	3.294	0.2	61.4	32.1
D 4b	9	1.098	0.01847	2.937	3.225	$1.48 \cdot 10^{-4}$	40.	20.9
D 5	10	1.079	0.01	2.989	3.225	$4.343 \cdot 10^{-4}$	40.	21.3

If technological requirements impose a maximal value of Q-factors (e.g., $Q_{max} = 20$ for active RC filters), Table 5.3 reveals that all six design alternatives fail. The design D4b is the best suboptimal solution.

An advanced design technique, the design D5 [47] with doubled poles [54], achieves the maximum Q-factor lower than 20. This is paid by increasing the filter order; the actual filter order is 12, which is much higher than the minimal order $n = 8$. Although the filter order has been increased, the implementation can be more cost effective [46, 55–58]; the lower tolerance components can be used, and the magnitude response of the implemented filter satisfies the specifications [46].

Table 5.3 Design space for $S_A = \{3000 \text{ Hz}, \ 3225 \text{ Hz}, \ 0.2 \text{ dB}, \ 40 \text{ dB}\}$

	Filter Order, n						
	8	9	10	11	12	13	16
ϵ_{min}	0.0758	0.0185	$\frac{3.687}{1000}$	$\frac{5.787}{10^4}$	$\frac{6.76}{10^5}$	$\frac{5.39}{10^6}$	$\frac{2.52}{10^{11}}$
ϵ_{max}	0.2171	0.2171	0.2171	0.2171	0.2171	0.2171	0.2171
$f_{p,min}$	2977	2937	2879	2800	2695	2556	1770
$f_{p,max}$	3092	3156	3189	3101	3206	3215	3225
$f_{s,min}$	3129	3066	3034	3117	3009	3004	3001
$f_{s,max}$	3250	3294	3360	3455	3590	3785	5467

We enlarge the design space by increasing the filter order from $n = 8$ to $n = 9$. The corresponding gain is plotted in Figs. 5.23–5.28, and the design results are summarized in Table 5.4. The Q-factor of the design D4b is 20.9 and is very close to the required maximal value ($Q = 20$).

Table 5.4 Q-factors for $S_A = \{3000\text{ Hz}, 3225\text{ Hz}, 0.2\text{ dB}, 40\text{ dB}\}$

	Filter Order, n								
	8	9	10	11	12	13	14	15	16
Q_{min}	22.1	20.9	20.3	20.0	19.8	19.6	19.6	19.5	19.5
Q_{max}	42.1	81.2	156	301	582	1121	2161	4167	8032

In practice, we choose the most suitable design, D, from the determined design space D_S. Thus, we can try to meet various technological requirements (maximal Q-factors, maximal element tolerances, and so forth) and advanced specifications (maximal group delay variation, maximal rise time, maximal overshoot in step response, maximal settling time).

It should be noticed that the design parameters of the design D5 belong to the design space D_S; D5 and its modification D5a [47] yield, also, elliptic function filters.

Suppose that n_{cheb} designates the order of the Chebyshev-type approximation meeting the specification. For $n = n_{cheb}$ the rational approximation function of D4a is practically equal to the polynomial Chebyshev type I approximation. On the other hand, for the same order $n = n_{cheb}$, the design D4b yields an inverse Chebyshev-type filter.

When the filter order is equal to the order of the Butterworth-type filter, the design D5 practically yields an allpole Butterworth-type filter. This means that the classic filter types—Chebyshev, inverse Chebyshev, and Butterworth—are just special cases of the elliptic function filters and are contained within the design space D_S.

5.6 VISUALIZATION OF DESIGN SPACE

Consider a lowpass filter from Section 5.5 specified by

$$S_A = \{F_p, F_s, A_p, A_s\} = \{3\,\text{kHz}, 3225\,\text{Hz}, 0.2\,\text{dB}, 40\,\text{dB}\}$$

with the characteristic-function-limit specification

$$S_K = \{F_p, F_s, K_p, K_s\} = \{3\,\text{kHz}, 3225\,\text{Hz}, 0.2171, 100\}$$

The minimal filter order has been calculated to be $n = 8$. Next, the range of ξ, ϵ, f_p, and f_s has been determined for $n \geq 8$.

The design subspaces for $n = 8$, $n = 9$, and $n = 13$ are shown in Figs. 5.29–5.31. Notice that the exact design subspaces are nonlinear continuous domains, but we draw rectangular blocks for the sake of simplicity.

Table 5.4 shows that the Q-factor varies from 15 to 156. It should be noticed that for the same specification S_A the MATLAB signal processing toolbox [1] offers an elliptic filter design with $Q = 42$, which might be inappropriate for practical analog implementations. Other classic approximations require very high orders compared to $n_{ellip} = 8$. The order of the Butterworth filter is extremely high ($n_{butt} = 85$), while

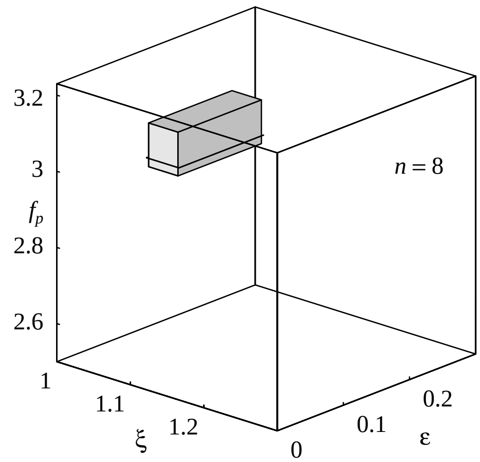

Figure 5.29 Design subspace for $n = 8$.

the order of the Chebyshev and inverse Chebyshev type is also high ($n_{cheb} = 18$) with high Q-factors ($Q_{cheb} = 46$ and $Q_{invcheb} = 20$). Generally, in analog filter design, filter orders that are significantly greater than n_{ellip} are unacceptable because they imply very high implementation complexity and cost.

The range of the design parameters ξ, ϵ, f_p, and f_s are shown in Figs. 5.32 and 5.33. The minimal filter order, $n = n_{min}$, implies a small range for design parameters, and the optimization of the filter behavior can be ineffective.

It is also worth noticing that increasing the filter order, $n > n_{min}$, does not necessarily lead to a better solution. However, there exists an advanced design strategy in which we choose $n > n_{min}$ and obtain robust and selective analog filters with better performance and reduced complexity. For example, the classic filter design failed to meet the specification in the case of an SC filter. However, a design based on our

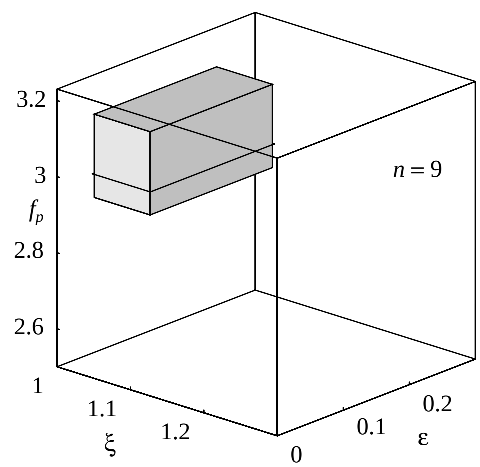

Figure 5.30 Design subspace for $n = 9$.

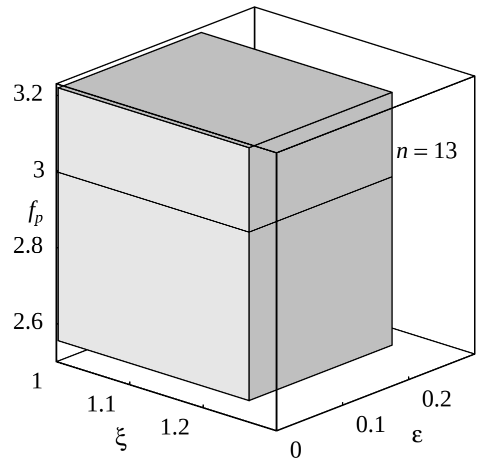

Figure 5.31 Design subspace for $n = 13$.

approach was practically implemented, and the measured filter characteristics showed that our advanced filter design was a successful one [46].

The group delay of the basic designs is plotted in Figs. 5.34 and 5.35.

The maximum group delay is obtained for the minimal transition designs, D3a and D3b, while the maximal transition designs, D4a and D4b, have lower group-delay variation. The design D3b has the minimal variation of the group delay in the passband, while the similar design D3a has the highest overall group-delay variation. The design D5, which is based on the minimal Q-factor design [47], has also a small variation of group delay in the passband.

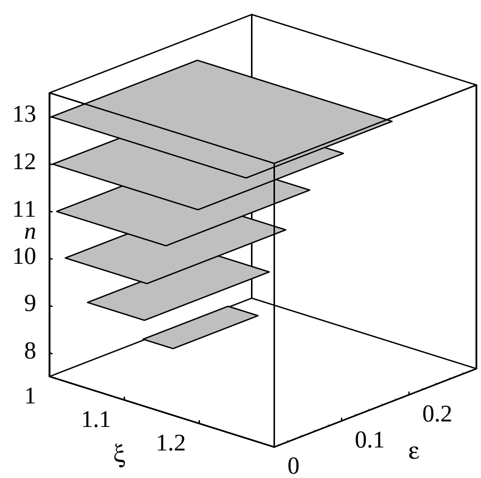

Figure 5.32 Design space of ξ, ϵ and n.

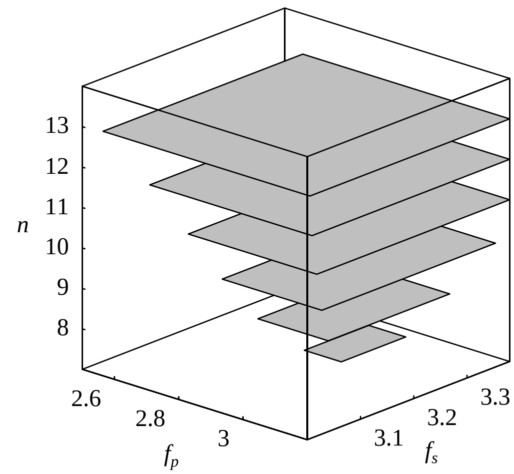

Figure 5.33 Design space of f_p, f_s and n.

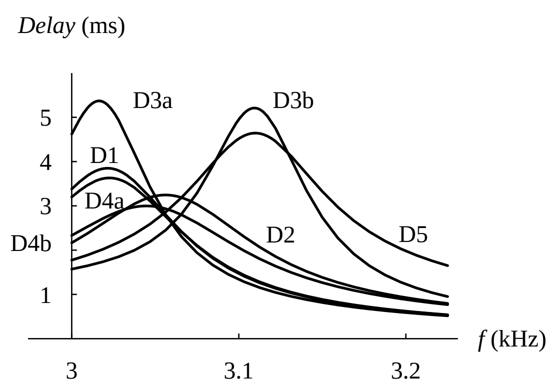

Figure 5.34 Group delay in the transition region.

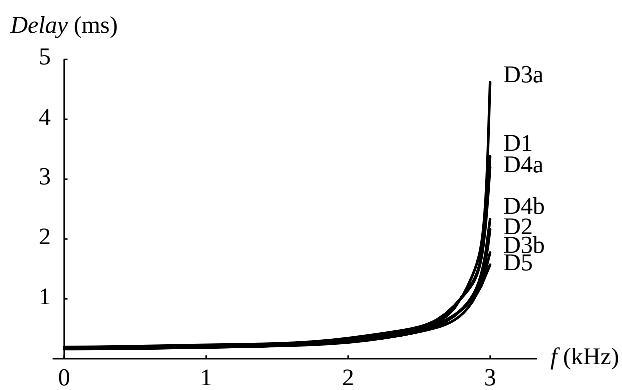

Figure 5.35 Group delay in the passband.

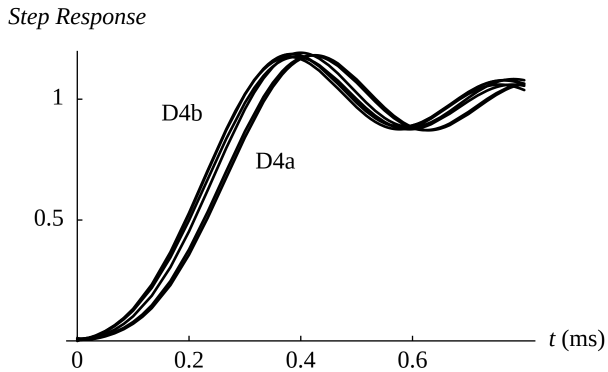

Figure 5.36 Step response of basic designs.

The step response of the basic designs is shown in Fig. 5.36. The shape of all responses is the same with approximately the same amount of overshoots (D3a and D3b have the largest overshoot). As we expect, D4b has the smallest time delay. However, the design D4a, which has also a good group-delay characteristic, has the worst time delay.

5.7 SC FILTER ADVANCED DESIGN EXAMPLE

Advanced filter design techniques can be efficiently exploited in designing robust and selective filters based on commercially available integrated switched capacitor (SC) universal building blocks. Application notes released by manufacturers rely on the classic filter design; often, these notes suggest standard modes of operation of the SC integrated chip that are suitable for different filter types and realizations.

In this section we modify a standard mode of operation of a universal SC filter to design very selective low sensitivity filters.

5.7.1 Introduction

The magnitude response of a high Q-factor second-order filter section is $2Q$ times more sensitive to the pole-magnitude variation than to the Q-factor variation. Traditionally, second-order filter sections (also called *biquads*) has been designed in such a way that the pole-magnitude sensitivities are at their theoretical minima. This means that the sensitivities to passive components (resistors and capacitors) have to be smaller than or equal to $\frac{1}{2}$ [59], [60]. Therefore, special care should be taken on pole-magnitude sensitivities.

The magnitude response of an SC filter is much more sensitive to the pole-magnitude variation than to the Q-factor variation as shown in Table 5.5.

Table 5.5 Approximate value of maximal passive sensitivities for $Q >> 1$

		$\left\|S_R^x\right\|$			$\max(\|S_R^{\|H\|}\|)$		
x	$\max(\|S_x^{\|H\|}\|)$	RC	SC 3a	SC 1 minQ	RC	SC 3a	SC 1 minQ
Q	1	Q	1	1	Q	1	1
ω_0	Q	$\frac{1}{2}$	$\frac{1}{2}$	0	$\frac{Q}{2}$	$\frac{Q}{2}$	0
ω_z	$\frac{1}{\omega_z^2-1}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{\omega_z^2-1}{2\omega_z^2}$	$\frac{1}{2(\omega_z^2-1)}$	$\frac{1}{2(\omega_z^2-1)}$	$\frac{1}{2}$
Q=20, $\omega_z = 1.078$ rad/s			$\max(\sum \|S_R^{\|H\|}\|)$		33.1	14.1	1.5

Passband edge frequency $\omega_p = 1$ rad/s.

Transfer function zero $\omega_z > \omega_p$.

Transfer function pole $\omega_0 > \omega_p$.

R—external resistor.

RC—second order active RC filter.

SC—switched-capacitor filter in mode 3a or mode 1 minQ.

Universal active SC filters can operate in seven modes. In almost all modes, except mode 1 and mode 4 [43, 44] the pole-magnitude sensitivities to passive components are $\frac{1}{2}$. In mode 1, the pole-magnitude sensitivity to passive components is 0. This means that the pole magnitude depends only on the clock frequency.

The pole Q-factor sensitivity to resistors, of universal SC biquads in mode 1, is equal to 1; it is Q times lower than the corresponding sensitivity of the best active RC filter with a single operational amplifier [59]. Nevertheless, mode 1 has been rarely recommended by manufacturers for practical realizations. The principal reason is that mode 1, usually, requires different clock frequencies for each biquad; thus its complexity increases.

Our target is to design a cost-effective filter with reduced complexity, and we want to use one clock frequency. If we prefer to use mode 1, we have to design a filter whose biquads have identical pole magnitudes.

One solution is the Butterworth filter because it has all poles on a circle; however, its order might be extremely high for very selective filter applications.

The minimal Q-factor filter design, D5, inherently has all poles on a circle; also, very selective specifications could be fulfilled with the order that is slightly higher than the minimal order. In addition, the design D5 features low Q-factors and reduced overall sensitivity. The maximal magnitude-response deviation, due to element tolerances, is in the transition region. Practically, the magnitude response in the passband is insensitive to variations of external resistor values.

5.7.2 Mode 1 Operation

The mode 1 biquad realization [44] is shown in Fig. 5.37.

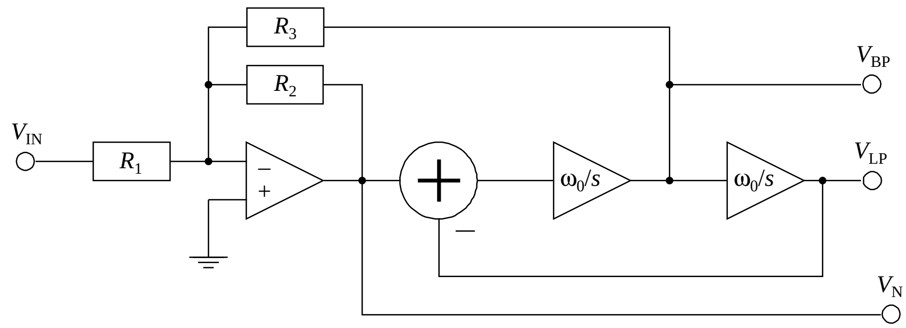

Figure 5.37 The mode 1 of an SC biquad.

The corresponding lowpass (LP), bandpass (BP), and notch (N) transfer functions are

$$H_{LP}(s) = \frac{V_{LP}}{V_{IN}} = -\frac{R_2}{R_1}\frac{\omega_0^2}{s^2 + \dfrac{R_2}{R_3}\omega_0 s + \omega_0^2} \tag{5.41}$$

$$H_{BP}(s) = \frac{V_{BP}}{V_{IN}} = -\frac{R_3}{R_1}\frac{\dfrac{R_2}{R_3}\omega_0 s}{s^2 + \dfrac{R_2}{R_3}\omega_0 s + \omega_0^2} \tag{5.42}$$

$$H_N(s) = \frac{V_N}{V_{IN}} = -\frac{R_2}{R_1}\frac{s^2 + \omega_0^2}{s^2 + \dfrac{R_2}{R_3}\omega_0 s + \omega_0^2} \tag{5.43}$$

where

$$\omega_0 = 2\pi\frac{f_{CLK}}{N} \tag{5.44}$$

$$N = 50 \qquad \text{or} \qquad N = 100 \tag{5.45}$$

The mode 1 operation is recommended for implementation of allpole lowpass and bandpass filters, such as Butterworth-, Chebyshev-, or Bessel-type filters [43, 44]. The mode 1 operation supports the highest clock frequencies because the input summing amplifier is outside the filter's resonant loop [44].

5.7.3 Modification to Mode 1 Operation

The lowpass minimal Q-factor design is essentially an elliptic design, and it yields imaginary transfer function zeros

$$\begin{aligned} s_z &= \pm j\omega_z \\ \omega_z &> \omega_0 \end{aligned} \tag{5.46}$$

and the biquad transfer function

$$H_{\min Q}(s) = -g \frac{s^2 + \omega_z^2}{s^2 + \dfrac{R_2}{R_3}\omega_0 s + \omega_0^2} \tag{5.47}$$

where g is a constant.

Can we modify the realization of Fig. 5.37 to satisfy the condition $\omega_z > \omega_0$? A possible modification is shown in Fig. 5.38, and we call it "mode 1 minQ". The biquad transfer function becomes

$$H_{\min Q}(s) = \frac{V_{N1}}{V_{IN}} = \frac{R_g R_2}{R_1 R_h} \frac{s^2 + \left(\omega_0 \sqrt{1 + \dfrac{R_h}{R_l}}\right)^2}{s^2 + \dfrac{R_2 \omega_0}{R_3} s + \omega_0^2} \tag{5.48}$$

and its zeros are determined by the clock frequency and the resistors R_h and R_l:

$$\omega_z = 2\pi \frac{f_{CLK}}{N} \sqrt{1 + \frac{R_h}{R_l}} \tag{5.49}$$

The pole Q-factor is

$$Q = \frac{R_3}{R_2} \tag{5.50}$$

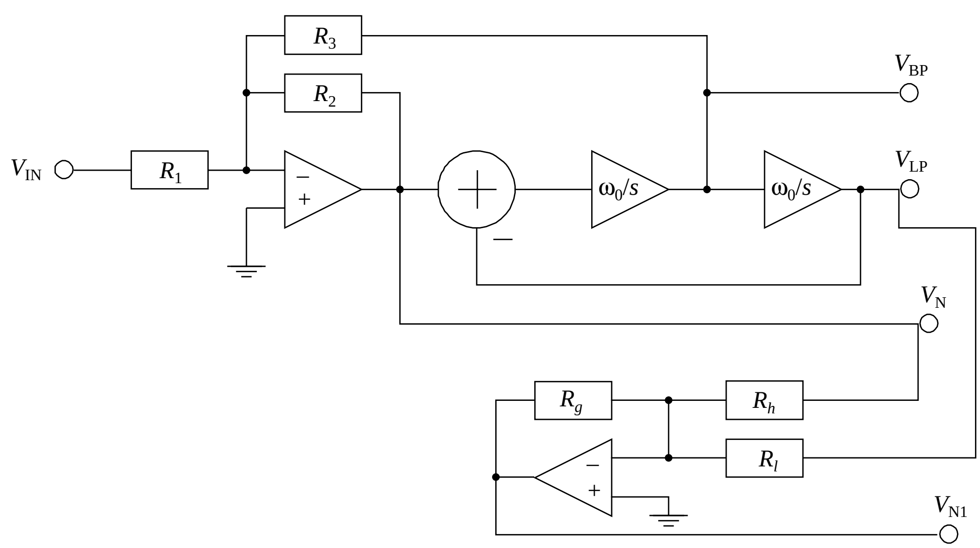

Figure 5.38 Mode 1 minQ operation.

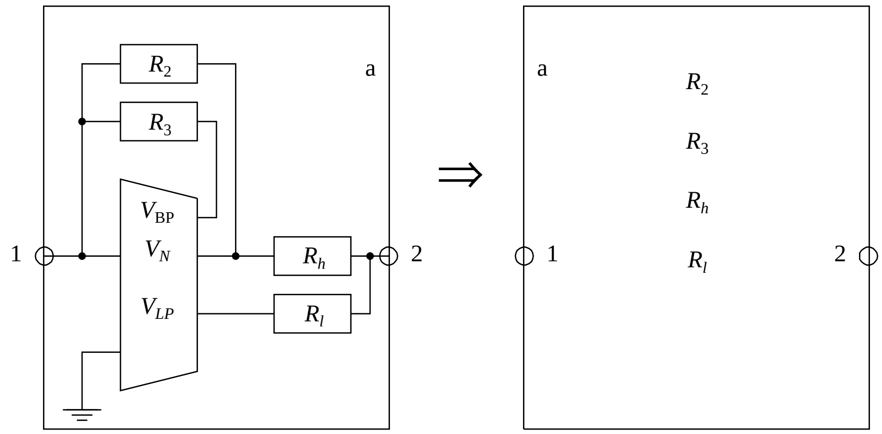

Figure 5.39 Second-order SC section (mode 1 minQ).

The performance of the implemented filter mostly depends on the accuracy of the clock frequency f_{CLK}, which is higher than the accuracy of passive components.

5.7.4 Cascaded Filter Realization

We implement a higher-order filter by cascading second-order sections. Each section is made out of an integrated SC building block and external resistors R_2, R_3, R_h, and R_l as shown in Figs. 5.39 and 5.40.

5.7.5 Practical Implementation

Consider a filter attenuation-limit specification

$$S_A = \{F_p = 2500 \text{ Hz}, \ F_s = 2690 \text{ Hz}, \ A_p = 0.5 \text{ dB}, \ A_s = 40 \text{ dB}\}$$

The manufacturer's application note [43, 44] suggests the mode 3a operation for the design of elliptic filters; the minimal filter order is $n_{\min} = 8$ (Table 5.6).

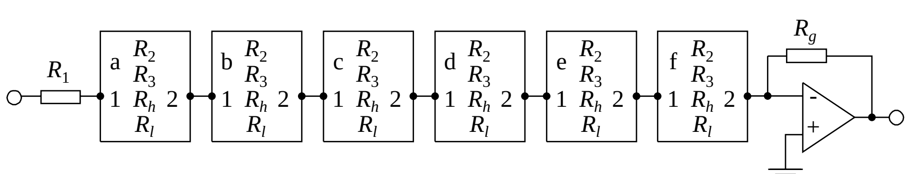

Figure 5.40 Filter implementation by cascading second-order SC sections.

Table 5.6 Resistor values for mode 3a

$R[\mathrm{k}\Omega]$, Mode 3a, n=8					
Section	R_2	R_3	R_4	R_h	R_l
a	10.9	10.0	24.7	262.	21.4
b	10.0	19.5	13.6	41.3	20.6
c	10.0	57.7	11.0	48.7	39.2
d	10.0	254.	10.4	10.9	10.0
$R_1 = 15.1\ \mathrm{k}\Omega$, $R_g = 67.8\ \mathrm{k}\Omega$					

We choose the mode 1 minQ operation and the order $n_{\min Q} = 12$. The element values are summarized in Table 5.7.

Table 5.7 Resistor values for mode 1 minQ

$R[\mathrm{k}\Omega]$, Mode 1 minQ, n=12				
Section	R_2	R_3	R_h	R_l
a	11.2	10.0	25.8	10.0
b	3.96	7.21	7.67	10.0
c	4.34	17.3	15.0	50.0
d	5.77	54.4	20.0	146.
e	4.40	151.	4.96	61.2
f	55.1	29.7	25.7	1.00
$R_1 = 10.0\ \mathrm{k}\Omega$, $R_g = 1.00\ \mathrm{k}\Omega$				

The sensitivity of the transfer function zeros, for mode 3a and mode 1 minQ operations, are shown in Table 5.8. The mode 1 minQ has significantly lower sensitivity.

Table 5.8 Sensitivity of transfer function zeros

	$\lvert S_{R_h}^{\lvert H\rvert}\rvert + \lvert S_{R_l}^{\lvert H\rvert}\rvert$		
Section/Mode	3a, $n = 8$	3a, $n = 12$	1 minQ, $n = 12$
a	0.08	0.04	0.04
b	0.87	0.35	0.26
c	2.97	1.11	0.53
d	5.86	2.52	0.72
e		4.50	0.82
f		6.17	0.86

The measured gain of both filters is shown in Figs. 5.41 and 5.42. The mode 1 minQ filter satisfies the specification. The mode 3a filter has larger passband ripple and violates the specification.

The measured passband attenuation is larger than the calculated attenuation. This disagreement is caused by the imperfections of the actual device (chip) that is used in

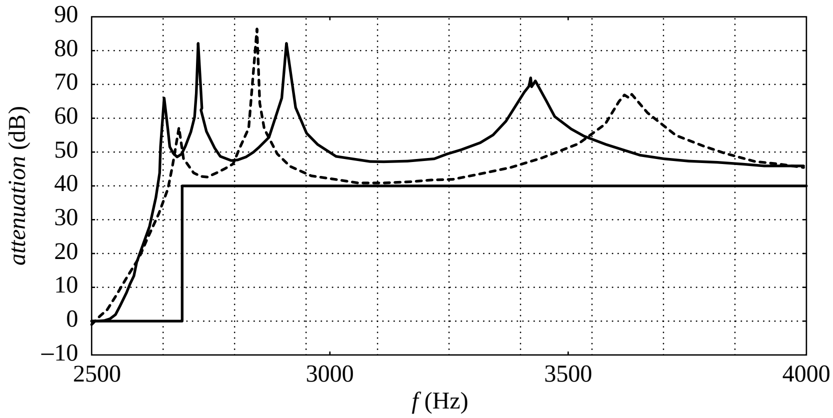

Figure 5.41 Measured attenuation in the stopband: mode 1 minQ (solid line), mode 3a (dashed line).

filter implementation. For a given f_{CLK} the frequency ω_0 varies from chip to chip, typically $\pm 0.2\%$.

5.7.6 Concluding Remarks

The transfer function poles of an integrated SC filter depend only on the clock frequency and are insensitive to variations of external resistors. The sensitivity of Q-factors to external resistors is equal to 1. The sensitivity of transfer function zeros to external resistors is lower than $\frac{1}{2}$.

Practically, our design is insensitive to external resistors. We can manufacture robust filters from standard resistors with looser tolerances. At the same time, we can lower the net price of the product and increase the product yield in mass production.

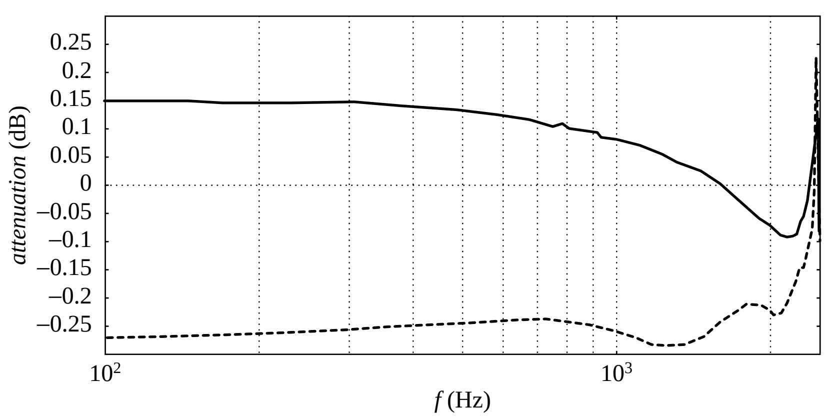

Figure 5.42 Measured attenuation in the passband: mode 1 minQ (solid line), mode 3a (dashed line).

PROBLEMS

5.1 Consider a lowpass filter with passband edge frequency at $F_p = 2000$ Hz, stopband edge frequency at $F_p = 2150$ Hz, maximum passband attenuation of $A_p = 0.2$ dB, and minimum stopband attenuation of $A_s = 40$ dB.

- **(a)** Find the magnitude limits specification, the magnitude tolerances specification, the magnitude ripple tolerances specification, and the gain limits specification.
- **(b)** Determine the minimum order of the filter. Determine the order of the Butterworth, Chebyshev type I, Chebyshev type II, and elliptic transfer function.
- **(c)** Determine the range of the selectivity factor for the orders $n = n_{\min}$ and $n = n_{\min+1}$.
- **(d)** Determine the range of the ripple factor for the orders $n = n_{\min}$ and $n = n_{\min+1}$.
- **(e)** Determine the range of the actual passband edge and actual stopband edge frequencies for the orders $n = n_{\min}$ and $n = n_{\min+1}$.

5.2 Design a minimal Q-factor lowpass filter with passband edge frequency at $F_p = 4000$ Hz, stopband edge frequency at $F_p = 4300$ Hz, maximum passband attenuation of $A_p = 0.2$ dB, and minimum stopband attenuation of $A_s = 40$ dB. Determine the minimum order of the filter, n_{mq}, and the filter order of the Butterworth transfer function, n_{but}. Plot the magnitude response of all minimal Q-factor lowpass filters for $n_{mq} \leq n \leq n_{but}$.

5.3 A filter shall be designed having a maximum passband attenuation $A_p = 0.1$ dB within the passband $0 \leq f \leq F_p = 10$ kHz and a minimum stopband attenuation $A_s = 40$ dB within the stopband $30 \leq f \leq F_s = 1000$ kHz. In addition, the filter shall meet the following:

- **(a)** The attenuation shall be equiripple in the passband and monotonic in the stopband, reaching the attenuation $a = 40$ dB at $f = 30$ kHz.
- **(b)** The attenuation shall be equiripple in the stopband and monotonic in the passband, reaching the attenuation $a = 0.1$ dB at $f = 10$ kHz.
- **(c)** The attenuation shall be monotonic in the passband and stopband, reaching the attenuation $a = 0.1$ dB at $f = 10$ kHz.
- **(d)** The attenuation shall be equiripple in the passband and stopband, reaching the attenuation $a = 0.1$ dB at $f = 10$ kHz.
- **(e)** The attenuation shall be equiripple in the passband and stopband, reaching the attenuation $a = 40$ dB at $f = 30$ kHz.

5.4 Design lowpass filters with passband edge frequency at $F_p = 2000$ Hz, stopband edge frequency at $F_p = 2150$ Hz, maximum passband attenuation of $A_p = 0.2$ dB, and minimum stopband attenuation of $A_s = 40$ dB. Determine pole Q-factors of the elliptic transfer function. Assume the filter orders $n = n_{\min}$, $n = n_{\min+1}$, $n = n_{\min+2}$, and $n = n_{\min+3}$. Determine pole Q-factors of the Butterworth, Chebyshev type I, and Chebyshev type II transfer functions.

5.5 Design lowpass filters with passband edge frequency at $F_p = 2000$ Hz, stopband edge frequency at $F_p = 2150$ Hz, maximum passband attenuation of $A_p = 0.2$ dB, and minimum stopband attenuation of $A_s = 40$ dB.

(a) Determine the minimum order of the filter, $n_{\min}$. Determine the minimum order of the Butterworth, n_{but}, Chebyshev type I, n_{c1}, Chebyshev type II, n_{c2}, and elliptic transfer function, n_e.

(b) Determine the range of the design parameters: selectivity factor, ripple factor, actual passband edge, and actual stopband edge frequencies, for $n_{\min} \le n \le n_{c1}$.

(c) Sketch the characteristic function of the designs D1, D2, D3a, D3b, D4a, and D4b, for $n_{\min} \le n \le n_{c1}$.

5.6 Design lowpass filters with passband edge frequency at $F_p = 3000$ Hz, stopband edge frequency at $F_p = 3225$ Hz, maximum passband attenuation of $A_p = 0.2$ dB, and minimum stopband attenuation of $A_s = 40$ dB.

(a) Determine the minimum order of the filter, $n_{\min}$. Determine the minimum order of the Butterworth transfer function, n_{but}.

(b) Sketch the characteristic function of the minimal Q-factor elliptic design (design D5) for $n_{\min} \le n \le n_{but}$.

(c) Compare the characteristic function of the minimal Q-factor elliptic design with the Butterworth design.

5.7 Design lowpass filters with passband edge frequency at $F_p = 3000$ Hz, stopband edge frequency at $F_p = 3225$ Hz, maximum passband attenuation of $A_p = 0.2$ dB, and minimum stopband attenuation of $A_s = 40$ dB.

(a) Determine the minimum order of the filter, $n_{\min}$. Determine the minimum order of the Chebyshev type I transfer function, n_{c1}.

(b) Sketch the characteristic function of the design D4a for $n_{\min} \le n \le n_{c1}$.

(c) Compare the characteristic function of the design D4a with the Chebyshev type I design.

5.8 Consider a lowpass filter specified by

$$S_K = \{F_p = 3 \text{ kHz}, \;\; F_s = 3.225 \text{ kHz}, \;\; K_p = 0.2171, \;\; K_s = 100\}$$

(a) Determine the minimum order of the filter, $n_{\min}$.

(b) Sketch the transfer function, step response, and group delay for designs D1, D2, D3, and D4, for $n = n_{\min}$.

(c) Compare the group delay and the rise time of the designs.

5.9 Design highpass filters with passband edge frequency at $F_p = 2150$ Hz, stopband edge frequency at $F_s = 2$ kHz, maximum passband attenuation of $A_p = 0.2$ dB, and minimum stopband attenuation of $A_s = 40$ dB.

(a) Determine the minimum order of the filter, $n_{\min}$. Determine the minimum order of the Butterworth, n_{but}, Chebyshev type I, n_{c1}, Chebyshev type II, n_{c2}, and elliptic transfer function, n_e.

(b) Determine the range of the design parameters: selectivity factor, ripple factor, actual passband edge, and actual stopband edge frequencies for $n_{\min} \le n \le n_{\min+2}$.

(c) Sketch the attenuation of the designs D1, D2, D3a, D3b, D4a, and D4b for $n_{\min} \le n \le n_{\min+2}$.

5.10 Design bandpass filters with passband edge frequencies at $F_{p1} = 3007$ Hz, $F_{p2} = 3217$ Hz, stopband edge frequencies at $F_{s1} = 3000$ Hz, $F_{s2} = 3225$ Hz, maximum passband attenuation of $A_p = 0.2$ dB, and minimum stopband attenuation of $A_s = 40$ dB.

(a) Determine the minimum order of the filter, $n_{\min}$. Determine the minimum order of the Butterworth, n_{but}, Chebyshev type I, n_{c1}, Chebyshev type II, n_{c2}, and elliptic transfer function, n_e.

(b) Determine the range of the design parameters: selectivity factor, ripple factor, actual passband edge, and actual stopband edge frequencies for $n_{\min} \le n \le n_{\min+2}$.

(c) Sketch the gain of the designs D1, D2, D3a, D3b, D4a, and D4b for $n_{\min} \le n \le n_{\min+2}$.

5.11 Design bandreject filters with stopband edge frequencies at $F_{s1} = 3007$ Hz, $F_{s2} = 3217$ Hz, passband edge frequencies at $F_{p1} = 3000$ Hz, $F_{p2} = 3225$ Hz, maximum passband attenuation of $A_p = 0.2$ dB, and minimum stopband attenuation of $A_s = 40$ dB.

(a) Determine the minimum order of the filter, $n_{\min}$. Determine the minimum order of the Butterworth, n_{but}, Chebyshev type I, n_{c1}, Chebyshev type II, n_{c2}, and elliptic transfer function, n_e.

(b) Determine the range of the design parameters: selectivity factor, ripple factor, actual passband edge, and actual stopband edge frequencies, for $n_{\min} \le n \le n_{\min+2}$.

(c) Sketch the attenuation of the designs D1, D2, D3a, D3b, D4a, and D4b for filter order $n_{\min} \le n \le n_{\min+2}$.

MATLAB EXERCISES

5.1 Write a MATLAB program that computes the magnitude limits specification, the magnitude tolerances specification, the magnitude ripple tolerances specification, and the gain limits specification. Assume the specification given in Problem 5.1. Determine the minimum order of the Butterworth, Chebyshev type I, Chebyshev type II, and elliptic transfer function. Plot the magnitude responses for filter order $n = n_{\min}$ and $n = n_{\min+1}$. Compute the range of the selectivity factor, the range of the ripple factor, and the range of the actual passband edge and actual stopband edge frequencies for filter order $n = n_{\min}$, $n = n_{\min+1}$, and $n = n_{\min+2}$.

5.2 Write a MATLAB program that plots the magnitude response of a minimal-Q-factor lowpass filter. Assume the specification given in Problem 5.2. Compute the magnitude of all poles and show that all poles of a minimal Q-factor transfer function lie on a circle. Plot poles and zeros of the transfer function for $n_{mq} \le n \le n_{but}$. Compute the minimum order of this filter, n_{mq}, and the minimal filter order of the Butterworth transfer function, n_{but}.

5.3 Write a MATLAB program that computes a lowpass transfer function with the specification given in Problem 5.4, the passband edge frequency at $F_p = 2000$ Hz, stopband edge frequency at $F_p = 2150$ Hz, maximum passband attenuation of $A_p = 0.2$ dB, and minimum stopband attenuation of $A_s = 40$ dB. Compute Q-factors of the elliptic transfer function for the filter orders $n = n_{\min}$, $n = n_{\min+1}$, $n = n_{\min+2}$, and $n = n_{\min+3}$. Compute the Q-factors of the Butterworth, Chebyshev type I, and Chebyshev type II transfer functions.

5.4 Write a MATLAB program that computes the characteristic function of the designs D1, D2, D3a, D3b, D4a, and D4b for filter order $n_{\min} \le n \le n_{c1}$. Assume the specification given in Problem 5.5. Compute the range of the selectivity factor, the ripple factor, the actual passband edge and actual stopband edge frequencies for filter order $n_{\min} \le n \le n_{c1}$.

5.5 Write a MATLAB program that computes the characteristic function of the minimal Q-factor elliptic and Butterworth transfer function for filter order $n_{\min} \leq n \leq n_{but}$. Assume the specification given in Problem 5.6.

5.6 Write a MATLAB program that computes the characteristic function of the design D4a and the Chebyshev type I transfer function for filter order $n_{\min} \leq n \leq n_{c1}$. Assume the specification given in Problem 5.7.

5.7 Write a MATLAB program that computes the transfer function, step response, and group delay for filter order $n = n_{\min}$. Assume the specification given in Problem 5.8.

5.8 Write a MATLAB program that computes the attenuation of the designs D1, D2, D3a, D3b, D4a, and D4b for highpass filter. Assume the specification given in Problem 5.9 and the filter order $n_{\min} \leq n \leq n_{\min+2}$.

5.9 Write a MATLAB program that plots the gain of the designs D1, D2, D3a, D3b, D4a, and D4b for bandpass filter. Assume the specification given in Problem 5.10 and the filter order $n_{\min} \leq n \leq n_{\min+2}$.

5.10 Write a MATLAB program that plots the gain of the designs D1, D2, D3a, D3b, D4a, and D4b for bandreject filter. Assume the specification given in Problem 5.11 and the filter order $n_{\min} \leq n \leq n_{\min+2}$.

■ *MATHEMATICA* EXERCISES

5.1 Write a *Mathematica* program that computes the magnitude limits specification, the magnitude tolerances specification, the magnitude ripple tolerances specification, and the gain limits specification. Assume the specification given in Problem 5.1. Determine the minimum order of the Butterworth, Chebyshev type I, Chebyshev type II, and elliptic transfer function. Plot the magnitude response for filter order $n = n_{\min}$ and $n = n_{\min+1}$. Compute the range of the selectivity factor, the range of the ripple factor, and the range of the actual passband edge and actual stopband edge frequencies for filter order $n = n_{\min}$, $n = n_{\min+1}$, and $n = n_{\min+2}$.

5.2 Write a *Mathematica* program that plots the magnitude response of a minimal-Q-factor lowpass filter. Assume the specification given in Problem 5.2. Compute the magnitude of all poles and show that all poles of a minimal Q-factor transfer function lie on a circle. Plot poles and zeros of the transfer function for $n_{mq} \leq n \leq n_{but}$. Compute the minimum order of the filter, n_{mq}, and the minimal filter order of the Butterworth transfer function, n_{but}.

5.3 Write a *Mathematica* program that computes lowpass transfer function with the specification given in Problem 5.4, the passband edge frequency at $F_p = 2000$ Hz, stopband edge frequency at $F_p = 2150$ Hz, maximum passband attenuation of $A_p = 0.2$ dB, and minimum stopband attenuation of $A_s = 40$ dB. Compute the Q-factors of elliptic transfer function for the filter orders $n = n_{\min}$, $n = n_{\min+1}$, $n = n_{\min+2}$, and $n = n_{\min+3}$. Compute the Q-factors of the Butterworth, Chebyshev type I, and Chebyshev type II transfer functions.

5.4 Write a *Mathematica* program that computes the characteristic function of the designs D1, D2, D3a, D3b, D4a, and D4b for filter order $n_{\min} \leq n \leq n_{c1}$. Assume the specification given in Problem 5.5. Compute the range of the selectivity factor, the ripple factor, the actual passband edge, and actual stopband edge frequencies for filter order $n_{\min} \leq n \leq n_{c1}$.

5.5 Write a *Mathematica* program that computes the characteristic function of the minimal Q-factor elliptic and Butterworth transfer function for filter order $n_{\min} \leq n \leq n_{but}$. Assume the specification given in Problem 5.6.

5.6 Write a *Mathematica* program that computes the characteristic function of the design D4a and the Chebyshev type I transfer function for filter order $n_{\min} \leq n \leq n_{c1}$. Assume the specification given in Problem 5.7.

5.7 Write a *Mathematica* program that computes the transfer function, step response, and group delay for filter order $n = n_{\min}$. Assume the specification given in Problem 5.8.

5.8 Write a *Mathematica* program that computes the attenuation of the designs D1, D2, D3a, D3b, D4a, and D4b for highpass filter. Assume the specification given in Problem 5.9 and the filter order $n_{\min} \leq n \leq n_{\min+2}$.

5.9 Write a *Mathematica* program that plots the gain of the designs D1, D2, D3a, D3b, D4a, and D4b for bandpass filter. Assume the specification given in Problem 5.10 and the filter order $n_{\min} \leq n \leq n_{\min+2}$.

5.10 Write a *Mathematica* program that plots the gain of the designs D1, D2, D3a, D3b, D4a, and D4b for bandreject filter. Assume the specification given in Problem 5.11 and the filter order $n_{\min} \leq n \leq n_{\min+2}$.