CHAPTER 4

CLASSIC ANALOG FILTER DESIGN

This chapter is intended to review the basics of classical analog filter design. Classification, salient properties, and sensitivity of transfer functions are given. The most important analog filter realizations are presented, including operational amplifier (op amp) active *RC*, switched-capacitor (SC), passive *RLC*, operational transconductance amplifier (OTA), and current-conveyor (CC) realizations. For each realization we provide complete design equations and procedures that make the design easily applicable to a broad variety of analog filter design problems. A detailed case study is given for the realization of various transfer functions.

4.1 INTRODUCTION TO ANALOG FILTERS

Analog filters are frequency-selective electrical circuits that are used to amplify or attenuate a single sinusoidal signal component or a portion of the signal frequency spectrum. The range of frequencies in which the sinusoidal signals are amplified or passed without considerable attenuation is called the *passband*. The frequency range in which the sinusoidal signals are significantly attenuated is called the *stopband*. The required minimum and maximum of the attenuation or amplification, along with the corresponding edge frequencies of the passbands and stopbands, are called the *specification*.

An *analog filter design* is a process in which we construct an electrical circuit that meets the given specification. The design starts with the specification and it consists of four basic steps: *approximation*, *realization*, *study of imperfections*, and *implementation* [16, XIII.65]. There is an infinite number of circuits that meet the specification; therefore, the filter design is by no means unique.

The example to follow details on the four design steps. A typical analog filter design is shown in Fig. 4.1.

Assume that we want to design a filter that passes sinusoidal signals over the frequency range $0 \leq f \leq F_p$, but attenuates the signals for $f \geq F_s > F_p$. This type of filter is called the *analog lowpass filter*. The frequency range $0 \leq f \leq F_p$ is the passband, and the range $F_s \leq f$ is the stopband; F_p is the *passband edge frequency* and F_s is the *stopband edge frequency*. We require that the attenuation in passband must not exceed A_p dB, and that the attenuation in stopband should be no less than A_s dB. Obviously, A_p is the *maximum passband attenuation*, and A_s is the *minimum stopband attenuation*. The four quantities that specify our lowpass filter requirements can be written in the form of the list $S = \{F_p, F_s, A_p, A_s\}$, which we simply call the *lowpass specification*.

In the first design step, the *approximation step*, we construct the filter transfer function $H(s)$, which is a rational function in the complex frequency s. The attenuation,

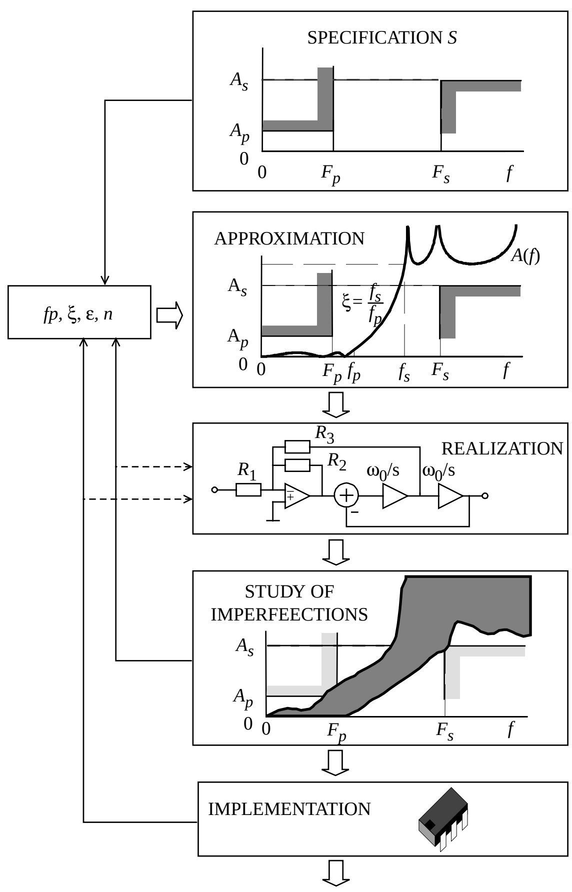

Figure 4.1 Classic filter design.

$A(f) = -20\log_{10}|H(j2\pi f)|$, must satisfy the specification S, that is

$$0 \le A(f) \le A_p, \qquad 0 \le f \le F_p \tag{4.1}$$

$$A_s \le A(f), \qquad F_s \le f \tag{4.2}$$

The function $A(f)$ is called the *attenuation approximation function*, or simply the *attenuation*.

Suppose that we choose the elliptic transfer function, known as the most selective function, which depends on four parameters: the order n, the actual passband edge frequency f_p, the actual stopband edge frequency f_s, and the ripple factor ϵ. It is convenient to define the ratio $\xi = f_s/f_p$, which is called the *selectivity factor*. We adjust the transfer-function parameters f_p, ξ, ϵ, and n to meet the specification; usually, we set $f_p = F_p$ and $\xi = F_s/F_p$. The maximal passband attenuation is controlled by the passband ripple factor ϵ, and we traditionally choose $\epsilon = \sqrt{10^{A_p/10} - 1}$. Generally, we prefer the minimum order n of the transfer function.

The *realization step* of an analog filter is the process of converting the transfer function into an electrical circuit; this circuit is sometimes called the *realization*. The designer is interested in realizations which are economical, simple, cheap, with small noise and distortion, and with high dynamic range and which are not seriously affected by small changes in the element values (tolerances, temperature and humidity variations, aging drift). In Fig. 4.1 a realization of a second-order universal SC filter is presented. Numerical values of the resistors are calculated from known $H(s)$.

In practice, the filter is implemented with nonideal elements and the designer must accomplish the *study of imperfections* which includes tolerance analysis and study of parasitics. If the specification can be satisfied only with high-precision expensive components, then the designer has to choose another transfer function $H(s)$ and reevaluate the realization or approximation step.

In the *implementation step* a device called the product prototype, also called the *implementation*, is constructed and tested. The cost of the mass production depends on the type of components, packaging, methods of manufacturing, testing, and tuning. The best implementation is a device with no need for tuning. If the requirements are not met, then the realization step (new circuit) or the approximation step (new f_p, ξ, ϵ, or n) must be redone.

The classical analog filters can be classified according to the frequency range they pass or reject:

- *Lowpass filter* passes sinusoidal signals over the range $0 \le f \le F_p$, but attenuates the signals for $f \ge F_s > F_p$.
- *Highpass filter* passes sinusoidal signals for $f \ge F_p$, but attenuates the signals over the range $0 \le f \le F_s < F_p$.
- *Bandpass filter* passes sinusoidal signals over the range $F_{p1} \le f \le F_{p2}$, but attenuates the signals for $0 \le f \le F_{s1} < F_{p1}$ and $f \ge F_{s2} > F_{p2}$.
- *Bandreject* or *bandstop filter* passes sinusoidal signals for $0 \le f \le F_{p1} < F_{s1}$ and $f \ge F_{p2} > F_{s2}$, but attenuates the signals over the range $F_{s1} \le f \le F_{s2}$.
- *Allpass filter* or *phase equalizer* passes sinusoidal signals without attenuation, and it shapes the phase response.

- *Lowpass-notch filter* rejects sinusoidal signals at frequencies $f \approx f_z$, but it passes signals at high frequencies ($f \gg f_z$) with some attenuation.
- *Highpass-notch filter* rejects sinusoidal signals at frequencies $f \approx f_z$, but it passes signals at low frequencies ($f \ll f_z$) with some attenuation.
- *Amplitude equalizer* or *bump filter* slightly amplifies or attenuates signals over a range of frequencies.

We mention here several, among many, excellent books on classical analog filter theory, analysis, and design: 15, 16, 23–34.

4.2 BASIC FILTER TRANSFER FUNCTIONS

The filter transfer function is a rational function in the complex frequency s and can be written in the form

$$H(s) = K \frac{(s - s_{z1})(s - s_{z2}) \cdots (s - s_{zm})}{\left(s - s_{p1}\right)\left(s - s_{p2}\right) \cdots \left(s - s_{pn}\right)} \tag{4.3}$$

where K is a real constant, $s_{z1}, \ldots, s_{zm}$ are zeros, and $s_{p1}, \ldots, s_{pn}$ are poles of the transfer function. Poles and zeros can be real or complex. Complex poles or zeros, s_i, occur in complex-conjugate pairs:

$$\begin{aligned} s_i &= \mathrm{Re}\,(s_i) + j\mathrm{Im}\,(s_i) \\ s_{i+1} &= \mathrm{Re}\,(s_i) - j\mathrm{Im}\,(s_i) \end{aligned} \tag{4.4}$$

The corresponding factors of the transfer function can be expressed as

$$(s - s_i)(s - s_{i+1}) = s^2 - (s_i + s_{i+1})s + s_i s_{i+1} \tag{4.5}$$

or can be presented by a second-order polynomial with real coefficients $a_i = -(s_i + s_{i+1})$ and $\omega_i^2 = s_i s_{i+1}$:

$$(s - s_i)(s - s_{i+1}) = s^2 + a_i s + \omega_i^2, \quad a_i = -2\ \mathrm{Re}\,(s_i), \qquad \omega_i = |s_i| \tag{4.6}$$

or equivalently

$$(s - s_i)(s - s_{i+1}) = s^2 + \frac{\omega_i}{Q_i} s + \omega_i^2 \tag{4.7}$$

In this book we prefer the form (4.7) because it is more significant from a practical point of view. The angular frequency ω_i is the *magnitude* of s_i. The quantity Q_i is called the *Q-factor* of s_i.

The Q-factor is used to characterize the ratio of the magnitude and the real part of a complex-conjugate pole or zero pair

$$Q_i = -\frac{|s_i|}{2\ \mathrm{Re}\,(s_i)} \tag{4.8}$$

Pole Q-factors of stable systems are always positive because $\mathrm{Re}(s_i) < 0$. Formally, when the imaginary part vanishes the Q-factor reaches its minimal value $1/2$.

A complex pole-zero pair can be represented by

$$H_i(s) = \frac{s^2 + \dfrac{\omega_{zi}}{Q_{zi}}s + \omega_{zi}^2}{s^2 + \dfrac{\omega_{pi}}{Q_{pi}}s + \omega_{pi}^2} \tag{4.9}$$

and is an example of the *second-order transfer function*, or *biquad*, for short.

To reduce the sensitivity of a transfer function with respect to deviations of the element values and to simplify the tuning process of the filter, it is preferable to realize the filter by a cascade of the first-order and second-order filter sections [30]. The cascade approach consists of realizing each of the biquads by an appropriate circuit and connecting these circuits in cascade. The overall transfer function, $H(s)$, could be expressed as a product of the first-order and second-order functions $H_i(s)$

$$H(s) = \prod_{i=1}^{N} H_i(s) \tag{4.10}$$

The advantage of the cascade approach is that the realization of a higher-order transfer function is reduced to the much simpler design of first-order and second-order filters. The individual low-order filters are isolated so that any change in one filter does not affect any other filter in the cascade. This property is useful for adjusting and testing the filter at the time of manufacture.

We characterize each second-order filter section by the frequencies of the magnitude response extremes and the 3-dB frequencies. The *3-dB frequencies*, denoted by $\omega_{3\text{dB}} = 2\pi f_{3\text{dB}}$, are frequencies at which the magnitude response $|H(j\omega)|$ is $\sqrt{2}$ times smaller than the magnitude response at some reference frequency $\omega_r = 2\pi f_r$ (3 dB only approximately corresponds to $\sqrt{2}$, strictly speaking it is $10^{3/20}$)

$$\frac{|H(j\omega_{3\text{dB}})|}{|H(j\omega_r)|} = \frac{1}{\sqrt{2}} \tag{4.11}$$

For example, for lowpass filters, the reference frequency is $\omega_r = 0$.

In filter design, we prefer to use the *normalized transfer function*, $H_n(s)$, defined by

$$H_n(s) = \frac{H(s)}{\max\limits_{\omega} |H(j\omega)|} \tag{4.12}$$

The transfer function can be obtained by scaling the normalized transfer function by a constant

$$H(s) = kH_n(s) \tag{4.13}$$

Quite generally, the normalization constant k can take any real value.

4.2.1 Second-Order Transfer Functions

In this section we analyze the properties of the basic second-order transfer functions. We examine the magnitude response $|H(j\omega)|$ for real positive angular frequencies ω. The angular frequencies of the magnitude response local extrema are designated by ω_e.

Lowpass Transfer Function. The second-order *lowpass transfer function* is defined as

$$H_{LP}(s) = \frac{\omega_p^2}{s^2 + \dfrac{\omega_p}{Q_p}s + \omega_p^2} \tag{4.14}$$

At high frequencies, $f \gg \omega_p/(2\pi)$, the magnitude response $|H_{LP}(j2\pi f)|$ decrease as f^2 and, thus, high-frequency sinusoidal signals are rejected.

The key properties of the lowpass transfer function are summarized below:

$$\begin{aligned}
&H_{LP}(0) = 1, && s = 0, \quad \omega = 0\\
&H_{LP}(s) \to 0, && s \to +\infty, \quad \omega \to +\infty\\
&|H_{LP}(j\omega_p)| = Q_p, && s = j\omega_p, \quad \omega = \omega_p
\end{aligned} \tag{4.15}$$

$$\begin{aligned}
&|H_{LP}(j\omega_e)| = \frac{Q_p}{\sqrt{1 - \dfrac{1}{4Q_p^2}}}, && \omega_e = \omega_p\sqrt{1 - \frac{1}{2Q_p^2}}\\
&|H_{LP}(j\omega_{3\text{dB}})| = \frac{1}{\sqrt{2}}, && \omega_{3\text{dB}} = \omega_p\sqrt{1 - \frac{1}{2Q_p^2} + \sqrt{1 - \left(1 - \frac{1}{2Q_p^2}\right)^2}}
\end{aligned} \tag{4.16}$$

The maximal value of the magnitude response is approximately equal to Q_p for $Q_p \gg 1$ as shown in Fig. 4.2:

$$\max_{\omega} |H_{LP}(j\omega)| = |H_{LP}(j\omega_e)| \approx Q_p, \quad \omega_e \approx \omega_p, \quad Q_p \gg 1 \tag{4.17}$$

This fact is very important for active filters. Suppose that the output voltage of operational amplifier must be from the range ± 1 V. The amplitude of the sinusoidal signal of frequency ω_e, at the input of the lowpass second-order filter, must be smaller than

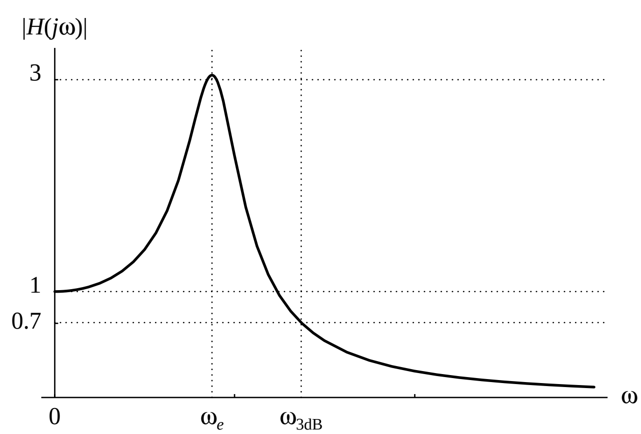

Figure 4.2 Magnitude of second-order lowpass transfer function: $Q_p = 3$ and $\omega_p = 0.9$.

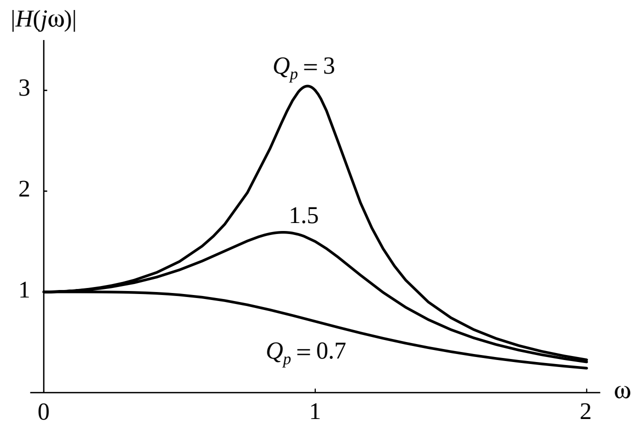

Figure 4.3 Magnitude of second-order lowpass transfer functions: $Q_p = 3$, 1.5, 0.7 and $\omega_p = 0.9$.

$1/Q_p$, so that, after filtering, the amplitude of the output signal remains within the prescribed range ± 1 V.

For $Q_p \leq 1/\sqrt{2}$ we find $\omega_e = 0$ as shown in Fig. 4.3.

The reference frequency for $\omega_{3\text{dB}}$, for lowpass filters, is $\omega_r = 0$.

Highpass Transfer Function. The second-order *highpass transfer function* is defined as

$$H_{HP}(s) = \frac{s^2}{s^2 + \dfrac{\omega_p}{Q_p}s + \omega_p^2} \tag{4.18}$$

The sinusoidal signals at very low frequencies, $f \ll \omega_p/(2\pi)$, are rejected while the high-frequency sinusoidal signals, $f \geq \omega_p/(2\pi)$, pass without attenuation.

The key properties of the highpass transfer function are summarized below:

$$\begin{aligned}
H_{HP}(0) &= 0, & s &= 0, \quad \omega = 0 \\
H_{HP}(s) &\to 1, & s &\to +\infty, \quad \omega \to +\infty \\
|H_{HP}(j\omega_p)| &= Q_p, & s &= j\omega_p, \quad \omega = \omega_p
\end{aligned} \tag{4.19}$$

$$\begin{aligned}
|H_{HP}(j\omega_e)| &= \frac{Q_p}{\sqrt{1 - \dfrac{1}{4Q_p^2}}}, & \omega_e &= \frac{\omega_p}{\sqrt{1 - \dfrac{1}{2Q_p^2}}} \\
|H_{HP}(j\omega_{3\text{dB}})| &= \frac{1}{\sqrt{2}}, & \omega_{3\text{dB}} &= \frac{\omega_p}{\sqrt{1 - \dfrac{1}{2Q_p^2} + \sqrt{1 - \left(1 - \dfrac{1}{2Q_p^2}\right)^2}}}
\end{aligned} \tag{4.20}$$

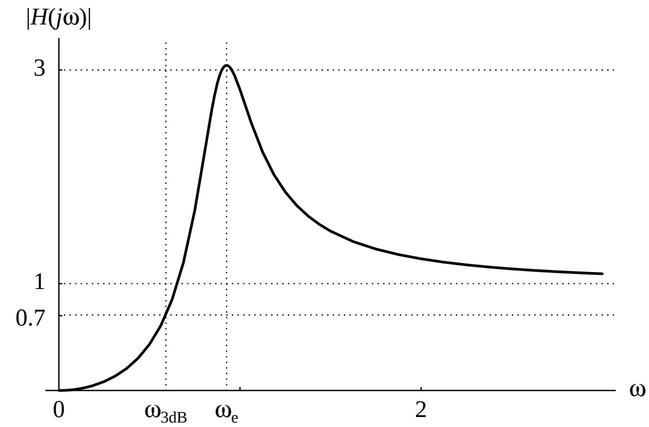

Figure 4.4 Magnitude of second-order highpass transfer function: $Q_p = 3$, and $\omega_p = 0.9$.

As in the case of lowpass filters, the maximal value of the magnitude response is approximately equal to Q_p, as shown in Fig. 4.4. For $Q_p \leq 1/\sqrt{2}$ we find that ω_e is at infinity, as shown in Fig. 4.5.

The reference frequency for ω_{3dB}, for highpass filters, is at infinity $\omega_r \to +\infty$.

Bandpass Transfer Function. The second-order *bandpass transfer function* is defined as

$$H_{BP}(s) = \frac{\dfrac{\omega_p}{Q_p}s}{s^2 + \dfrac{\omega_p}{Q_p}s + \omega_p^2} \tag{4.21}$$

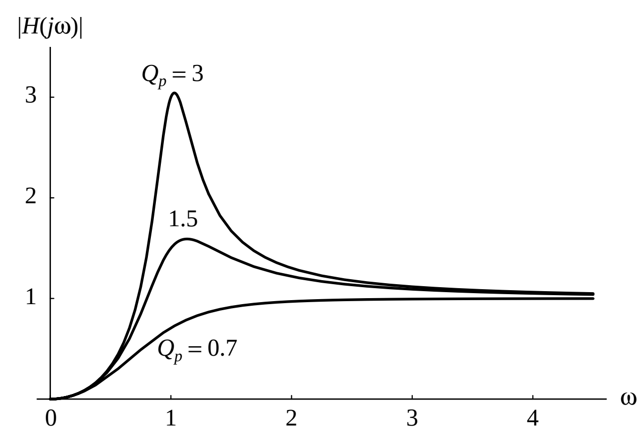

Figure 4.5 Magnitude of second-order highpass transfer functions: $Q_p = 3$, 1.5, 0.7 and $\omega_p = 0.9$.

The key properties of the bandpass transfer function are summarized below:

$$\begin{aligned}
&H_{BP}(0) = 0, && s = 0, \quad \omega = 0 \\
&H_{BP}(s) \to 0, && s \to +\infty, \quad \omega \to +\infty \\
&|H_{BP}(j\omega_p)| = 1, && s = j\omega_p, \quad \omega = \omega_p \\
&|H_{BP}(j\omega_e)| = 1, && \omega_e = \omega_p
\end{aligned} \tag{4.22}$$

The angular frequency at which the magnitude response reaches its maximum, ω_e, is equal to the pole magnitude, ω_p, and is sometimes called the *resonant angular frequency*, or the *central angular frequency*.

$$\begin{aligned}
&|H_{BP}(j\omega_{low,3\text{dB}})| = \frac{1}{\sqrt{2}}, \qquad \omega_{low,3\text{dB}} = \omega_p \sqrt{1 + \frac{1}{2Q_p^2} - \frac{1}{Q_p}\sqrt{1 + \left(\frac{1}{2Q_p}\right)^2}} \\
&|H_{BP}(j\omega_{high,3\text{dB}})| = \frac{1}{\sqrt{2}}, \qquad \omega_{high,3\text{dB}} = \omega_p \sqrt{1 + \frac{1}{2Q_p^2} + \frac{1}{Q_p}\sqrt{1 + \left(\frac{1}{2Q_p}\right)^2}}
\end{aligned} \tag{4.23}$$

Second-order bandpass filters pass sinusoidal signals from the band of frequencies $\omega_{low,3\text{dB}}/(2\pi) < f < \omega_{high,3\text{dB}}/(2\pi)$ with insignificant attenuation, but reject sinusoidal signals whose frequencies are on either side of this band. The *3-dB bandwidth* of a bandpass filter is defined as

$$\omega_{BW} = \omega_{high,3\text{dB}} - \omega_{low,3\text{dB}} \tag{4.24}$$

and the *relative bandwidth* is

$$\text{relative bandwidth} = \frac{\omega_{BW}}{\omega_p}$$

The maximum of the magnitude response is 1. The 3-dB bandwidth is affected by Q_p (Fig. 4.6). Higher Q-factors produce narrower bandwidths (Fig. 4.7).

The reference frequency for $\omega_{3\text{dB}}$, for bandpass filters, is $\omega_r = \omega_p$.

Bandreject Transfer Function. The second-order *bandreject transfer function* is defined as

$$H_{BR}(s) = \frac{s^2 + \omega_p^2}{s^2 + \dfrac{\omega_p}{Q_p}s + \omega_p^2} \tag{4.25}$$

The key properties of the bandreject transfer function are summarized below:

$$\begin{aligned}
&H_{BR}(0) = 1, && s = 0, \quad \omega = 0 \\
&H_{BR}(s) = 1, && s \to +\infty, \quad \omega \to +\infty \\
&|H_{BR}(j\omega_p)| = 0, && s = j\omega_p, \quad \omega = \omega_p
\end{aligned} \tag{4.26}$$

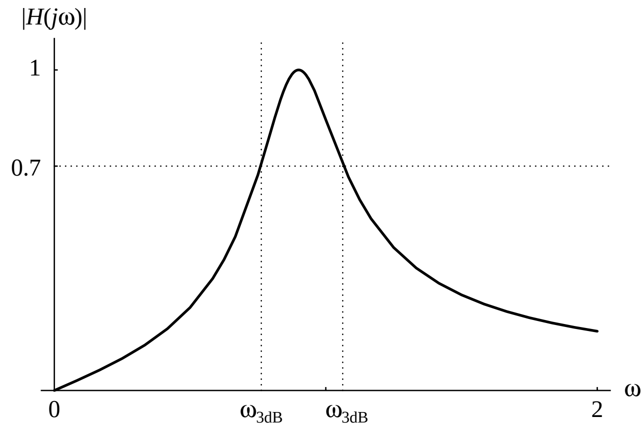

Figure 4.6 Magnitude of second-order bandpass transfer function: $Q_p = 3$ and $\omega_p = 0.9$.

$$|H_{BR}(j\omega_{low,3\text{dB}})| = \frac{1}{\sqrt{2}}, \qquad \omega_{low,3\text{dB}} = \omega_p \sqrt{1 + \frac{1}{2Q_p^2} - \frac{1}{Q_p}\sqrt{1 + \left(\frac{1}{2Q_p}\right)^2}}$$

$$|H_{BR}(j\omega_{high,3\text{dB}})| = \frac{1}{\sqrt{2}}, \qquad \omega_{high,3\text{dB}} = \omega_p \sqrt{1 + \frac{1}{2Q_p^2} + \frac{1}{Q_p}\sqrt{1 + \left(\frac{1}{2Q_p}\right)^2}} \tag{4.27}$$

Bandreject filters reject sinusoidal signals from the band of frequencies $\omega_{low,3\text{dB}}/(2\pi) < f < \omega_{high,3\text{dB}}/(2\pi)$, but they pass signals whose frequencies are on either side of this band. The *3-dB bandwidth* of a bandreject filter is defined as

$$\omega_{BW} = \omega_{high,3\text{dB}} - \omega_{low,3\text{dB}} \tag{4.28}$$

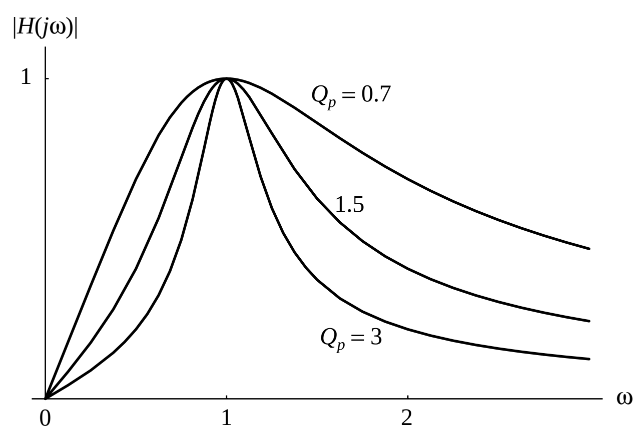

Figure 4.7 Magnitude of second-order bandpass transfer functions: $Q_p = 3$, 1.5, 0.7 and $\omega_p = 0.9$.

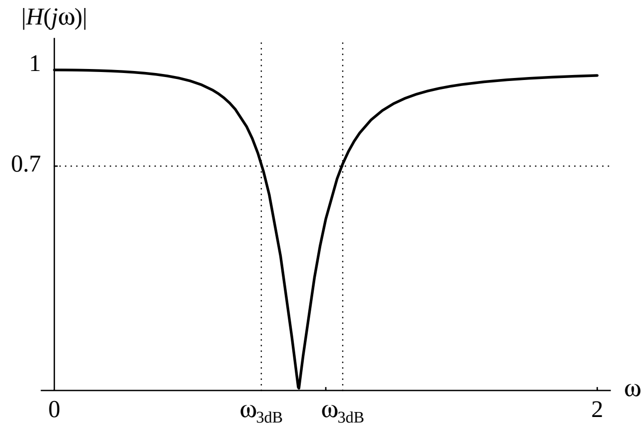

Figure 4.8 Magnitude of second-order bandreject transfer function: $Q_p = 3$ and $\omega_p = 0.9$.

The maximum of the magnitude response is 1 (Fig. 4.8), and the magnitude response reaches its minimum at $\omega = \omega_p$. The 3-dB bandwidth is affected by Q_p (Fig. 4.9); higher Q-factors produce narrower bandwidths.

The reference frequency for $\omega_{3\text{dB}}$, for bandreject filters, is $\omega_r = 0$.

Lowpass-Notch and Highpass-Notch Transfer Function. The second-order *lowpass-notch* and *highpass-notch transfer functions* are defined as

$$H_{LPN}(s) = \frac{s^2 + \omega_z^2}{s^2 + \dfrac{\omega_p}{Q_p}s + \omega_p^2}, \quad \omega_z^2 > \omega_p^2$$
$$H_{HPN}(s) = \frac{s^2 + \omega_z^2}{s^2 + \dfrac{\omega_p}{Q_p}s + \omega_p^2}, \quad \omega_z^2 < \omega_p^2 \tag{4.29}$$

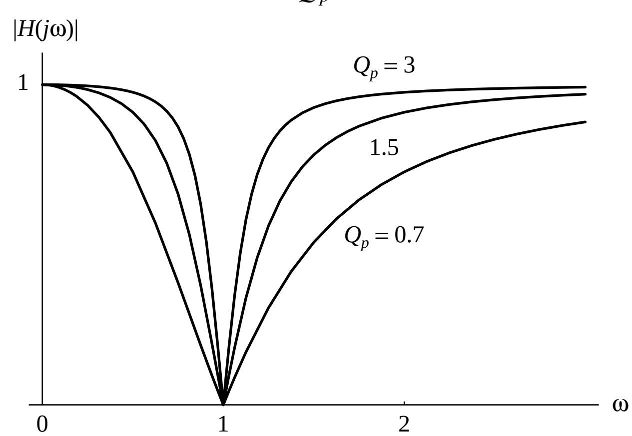

Figure 4.9 Magnitude of second-order bandreject transfer functions: $Q_p = 3$, 1.5, 0.7 and $\omega_p = 0.9$.

Lowpass-notch filters reject sinusoidal signals whose frequencies are $f \approx \omega_z/(2\pi)$, but they pass signals at high frequencies ($f \gg \omega_z/(2\pi)$) with some attenuation. Highpass-notch filters reject sinusoidal signals whose frequencies are $f \approx \omega_z/(2\pi)$, but they pass signals at low frequencies ($f \ll \omega_z/(2\pi)$) with some attenuation.

The key properties of the lowpass-notch and highpass-notch transfer function are summarized below:

$$
\begin{aligned}
&H_{LPN}(0) = H_{HPN}(0) = \frac{\omega_z^2}{\omega_p^2}, && s = 0, \quad \omega = 0\\
&H_{LPN}(s) = H_{HPN}(s) = 1, && s \to +\infty, \quad \omega \to +\infty\\
&|H_{LPN}(j\omega_p)| = |H_{HPN}(j\omega_p)| = \left|Q_p\left(-1 + \frac{\omega_z^2}{\omega_p^2}\right)\right|, && s = j\omega_p, \quad \omega = \omega_p\\
&H_{LPN}(j\omega_z) = H_{HPN}(j\omega_z) = 0, && s = j\omega_z, \quad \omega = \omega_z
\end{aligned}
\tag{4.30}
$$

The maximum of the magnitude response occurs at ω_e and is given below:

$$
\begin{aligned}
|H_{LPN}(j\omega_e)| = |H_{HPN}(j\omega_e)| &= \frac{2Q_p\omega_z^2}{\omega_p^2}\sqrt{\frac{Q_p^2 + (1 - 2Q_p^2)\dfrac{\omega_p^2}{\omega_z^2} + Q_p^2\dfrac{\omega_p^4}{\omega_z^4}}{4Q_p^2 - 1}}\\
\omega_e &= \omega_p\sqrt{\frac{2Q_p^2\omega_p^2 + \omega_z^2 - 2Q_p^2\omega_z^2}{-\omega_p^2 + 2Q_p^2\omega_p^2 - 2Q_p^2\omega_z^2}}
\end{aligned}
\tag{4.31}
$$

The reference frequency for $\omega_{3\mathrm{dB}}$, for notch filters, is at infinity $\omega_r \to +\infty$, as shown in Fig. 4.10.

The maximum of the magnitude response in terms of ω_p/ω_z is plotted in Fig. 4.11; notice that it reaches the minimum for $\omega_p = \omega_z$.

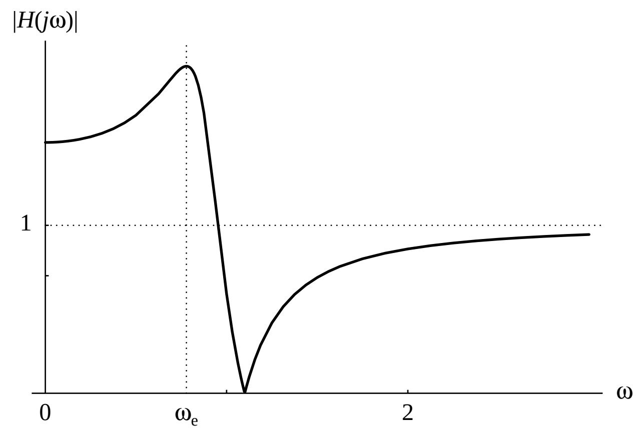

Figure 4.10 Magnitude of second-order lowpass-notch transfer function, $Q_p = 3$, $\omega_p = 0.9$, and $\omega_z = 1.1$.

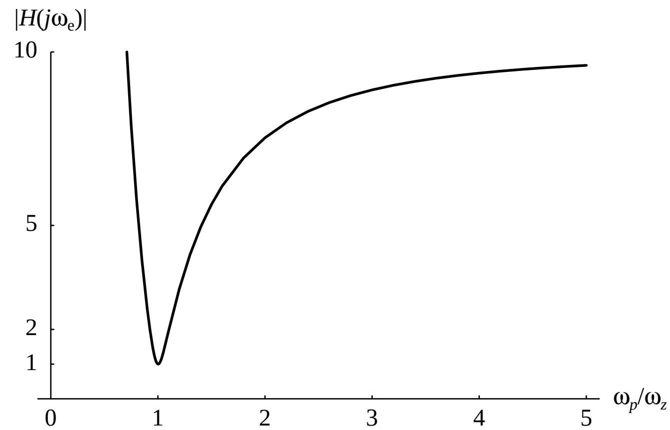

Figure 4.11 The maximum of magnitude of second-order notch transfer function, $Q_p = 10$.

Allpass Transfer Function. The second-order *allpass transfer function* is defined as

$$H_{AP}(s) = \frac{s^2 - \dfrac{\omega_p}{Q_p}s + \omega_p^2}{s^2 + \dfrac{\omega_p}{Q_p}s + \omega_p^2} \tag{4.32}$$

The magnitude of the allpass transfer function is constant for all ω:

$$|H_{AP}(j\omega)| = 1 \tag{4.33}$$

Thus far, we have discussed the magnitude response only. However, for allpass transfer functions the phase response is of key interest. Allpass filters can be used to modify the phase responses of filters without changing their magnitude responses. Usually, they are used to satisfy magnitude and phase specifications simultaneously.

The phase response of second-order allpass filters is shown in Fig. 4.12. If the Q-factor is smaller, then the phase response of an allpass filter is more linear.

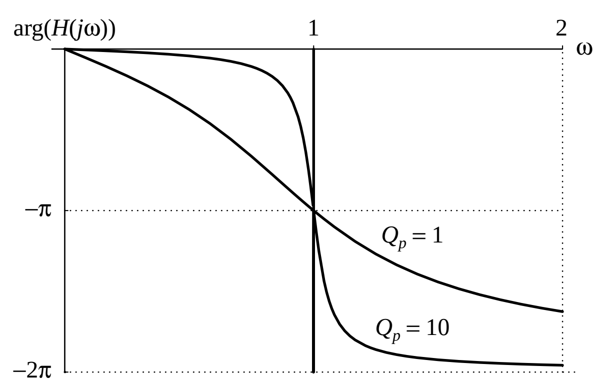

Figure 4.12 Phase of second-order allpass transfer functions, $Q_p = 10$, 1, and $\omega_p = 1$.

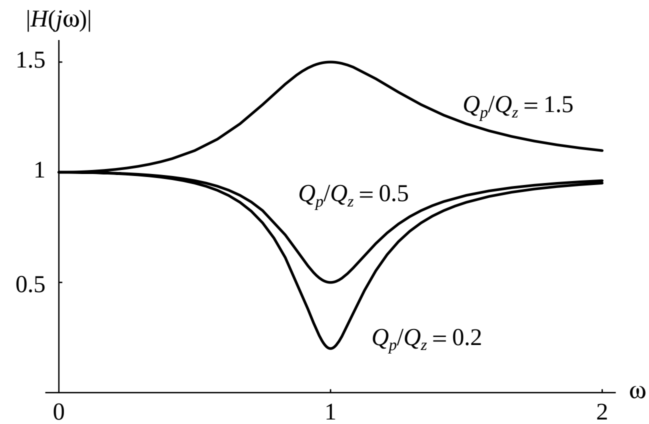

Figure 4.13 Magnitudes of second-order amplitude equalizer transfer function, $\omega_p = 1$.

The zeros of an allpass transfer function are mirror images of the transfer function poles, in the complex plane, with respect to the imaginary axis.

Amplitude Equalizer Transfer Function. *Amplitude equalizer* or *bump filter* slightly amplifies (boosts) or attenuates signals over a range of frequencies and has the transfer function

$$H_{AE}(s) = \frac{s^2 + \dfrac{\omega_p}{Q_z}s + \omega_p^2}{s^2 + \dfrac{\omega_p}{Q_p}s + \omega_p^2} \tag{4.34}$$

The key properties of the amplitude equalizer transfer function are summarized below:

$$\begin{aligned} H_{AE}(0) &= 1, && s = 0, \quad \omega = 0 \\ H_{AE}(s) &= 1, && s \to +\infty, \quad \omega \to +\infty \\ H_{AE}(j\omega_p) &= \frac{Q_p}{Q_z}, && s = j\omega_p, \quad \omega = \omega_p \end{aligned} \tag{4.35}$$

The magnitude response of the amplitude equalizer is shown in Fig. 4.13.

4.3 DECOMPOSITION OF TRANSFER FUNCTIONS

Any higher-order transfer function can be expressed as a product of the first-order and the second-order transfer functions. In practice, when designing a filter, we prefer transfer functions with complex-conjugate poles; however, if the transfer-function order is odd, we prefer transfer functions with only one real pole:

$$H_{even}(s) = \prod_i \frac{b_{2,i}s^2 + b_{1,i}s + b_{0,i}}{s^2 + \frac{\omega_{pi}}{Q_{pi}}s + \omega_{pi}^2} \tag{4.36}$$

$$H_{odd}(s) = \frac{b_1 s + b_0}{s + \omega_p} \prod_i \frac{b_{2,i}s^2 + b_{1,i}s + b_{0,i}}{s^2 + \frac{\omega_{pi}}{Q_{pi}}s + \omega_{pi}^2} \tag{4.37}$$

The first-order and the second-order transfer functions are realized separately. Next, the transfer function is realized by cascading these low-order filter sections. The cascade design is attractive for various reasons:

- The filter design is straightforward.
- The case study for selecting the appropriate low-order filter realizations is facilitated.
- The effort for testing and tuning the filter is reduced.

4.4 POLE-ZERO PAIRING

The *pole-zero pairing* involves (a) a decomposition of the numerator and the denominator of the transfer function into products of constant terms, and the first-order and the second-order functions in s and, then, (b) constructing the first-order and the second-order rational functions in s by pairing the numerator and denominator terms.

The pole-zero pairing is not unique and depends on implementation, as well as on the selected transfer function. A detailed analysis of this topic can be found in reference 29.

The most frequently used procedure is pairing the poles with higher Q-factors with zeros that are as close as possible to the poles. This pairing can achieve the maximal dynamic range of the second-order sections.

The most important criteria that are used in practice, as criteria for pole-zero pairing, are as follows:

1. *Maximal dynamic range*: A figure-of-merit of the dynamic range of the second-order filter section can be determined as a ratio of (a) the maximum magnitude response computed over the whole frequency range to (b) the minimum magnitude response in the passband. This criterion is of importance when large signals are processed.
2. *Maximal signal-to-noise ratio*: The maximal signal-to-noise ratio that does not drive the filter out of the linear mode of operation, assuming that the noise is generated within the active device.
3. *Minimal inband losses*: The minimum magnitude response of a filter section, computed over the passband, defines the maximum inband loss. Our goal is to find pole-zero pairing that keeps this loss as low as possible.
4. *Minimal sensitivity*: The overall sensitivity of the filter can be reduced by appropriate pole-zero pairing.

The choice of the scaling factors of the first-order and the second-order sections is called the *gain distribution*. There exist procedures for optimizing the gain distribution for maximal dynamic range or for minimal sensitivity [29].

4.5 OPTIMUM CASCADING SEQUENCE

After a higher-order transfer function is decomposed into, say m, first-order and second-order functions, with appropriate scaling factors, the designer has to choose the *optimal cascading sequence*. The number of possible cascade realizations is $m!$, where m is, also, the number of cascaded filter sections.

For example, a sixth-order transfer function can be decomposed into the product $H(s) = H_1(s)H_2(s)H_3(s)$, and realized as a cascade of three second-order sections. The number of possible realizations is $m! = 3! = 6$ as shown in Fig. 4.14.

Frequently, for the maximal dynamic range, the optimal sequence of the second-order filter sections is the sequence in which the preceding section has lower Q-factor than the following section.

In some cases, some other criteria may be more important in choosing the optimal cascading sequence. For example, if the input signal contains an undesired signal with very large amplitude, this signal has to be filtered out by the very first section (the first section should have a transfer function zero at the frequency of the undesired signal). Thus, we avoid possible distortion of the desired signal at the remaining sections.

Special care must be taken when we filter pulse signals that contain very high frequency components. These signals may pass through the filter (e.g., through the passive components) without attenuation because active devices do not have the expected or assumed characteristics at higher frequencies (e.g., operational amplifiers can have no gain at very high frequencies). Therefore, the first filter section must be carefully chosen and, sometimes, only passive components (such as resistors and capacitors) should be used to ensure required attenuation at very high frequencies.

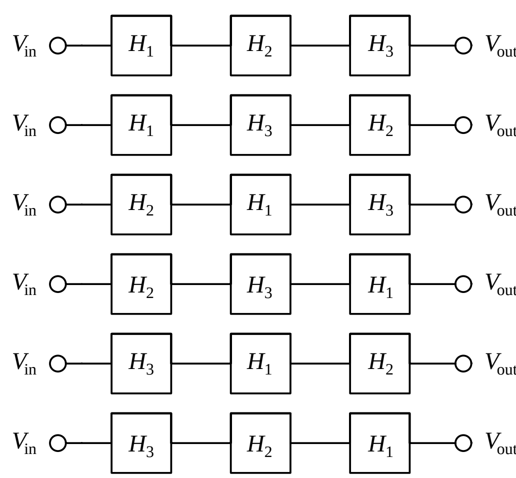

Figure 4.14 All cascade realizations of sixth-order transfer function.

4.6 SENSITIVITY

4.6.1 Basic Definitions

Performances of manufactured filters cannot be guaranteed to correspond exactly to the designed filter performances. The differences are due to component imperfections such as the following:

- The element value is not an exact number, it is always within prescribed tolerances; for the nominal element value x_{nom} and tolerance x_t (%) the actual value is from the range $x_{nom} - x_t/100 < x < x_{nom} + x_t/100$.
- Ambient variations (temperature, humidity) and chemical changes due to aging change the element value $x_{nom} - \triangle x_1 < x < x_{nom} + \triangle x_2$, usually, $\triangle x_1 \neq \triangle x_2$.

Many circuit realizations can be used to satisfy the prescribed filter transfer function. If we assume ideal circuit elements (perfect components), various realizations will exhibit the same performances. In practice, real components are imperfect and their values deviate from the nominal values; therefore, performances of the manufactured filter differ from the performances of the filter with perfect components.

Before the filter is manufactured, the effects of manufacturing tolerances and component imperfections have to be analyzed. Such an analysis allows the designer to predict variations of the filter performances and to predict the production yield. The *yield* is the ratio of (a) the number of manufactured filters satisfying the specification to (b) the total number of manufactured filters. Obviously, a high yield is desirable for profitability.

The simplest way to predict the yield is to use the concept of network sensitivity, assuming that the component changes are small.

The *single-parameter relative sensitivity* of a function $F = F(x_1, \ldots, x_i, \ldots, x_n)$, due to a change in quantity x_i, is

$$S_{x_i}^F = \frac{x_i}{F}\frac{\partial F}{\partial x_i} \tag{4.38}$$

and, for a small change in x_i, the relative variation of the function F is expected to be

$$\frac{\Delta F}{F} = S_{x_i}^F \frac{\Delta x_i}{x_i} \tag{4.39}$$

where $\frac{\Delta x_i}{x_i}$ is called the *variability* of x.

For simpler notation we use $S(F, x_i)$ instead of $S_{x_i}^F$:

$$S(F, x_i) = S_{x_i}^F \tag{4.40}$$

The expected *relative variation* in function $F = F(x_1, \dots, x_i, \dots, x_n)$, due to the changes in all quantities $x_1, \dots, x_i, \dots, x_n$, is given by

$$\frac{\Delta F}{F} = \sum_{i=1}^{n} S_{x_i}^{F} \frac{\Delta x_i}{x_i} \tag{4.41}$$

The function $F(x_1, \dots, x_i, \dots, x_n)$ can be a transfer function, the magnitude of a transfer function, the filter attenuation, the group delay, or any other function derived from the transfer function, such as a pole of the transfer function, the Q-factor of a pole, or a transfer function coefficient.

If $S_{x_i}^{F} = 1$, then a 1% change in the quantity x_i will cause a 1% change in the function F. When a sensitivity is zero, $S_{x_i}^{F} = 0$, then any change in x_i will not affect the function F.

In practice, the upper limit of the relative variation $\frac{\Delta F}{F}$ is estimated by three methods:

- The *worst-case* method gives the absolute relative variation

$$\left.\frac{\Delta F}{F}\right|_{\text{worst case}} = \sum_{i=1}^{n} \left| S_{x_i}^{F} \frac{\Delta x_i}{x_i} \right| \tag{4.42}$$

- The *Schoeffler criterion* method is based on the fomula

$$\left.\frac{\Delta F}{F}\right|_{\text{Schoeffler}} = \sqrt{\sum_{i=1}^{n} \left| S_{x_i}^{F} \frac{\Delta x_i}{x_i} \right|^2} \tag{4.43}$$

- The *Monte Carlo* method relies on extensive simulation of the filter realization (circuit) with randomly chosen element values. The performances are interpreted statistically.

The worst-case variation is a rather pessimistic figure-of-merit, assuming that all changes are at their extreme. The *Schoeffler* variation is more realistic, and it is closer to the variation obtained by statistical computations. The most preferable method is the *Monte Carlo* method. It evaluates filter performances for random combination of element values. We can use different statistical distributions of element values to model component imperfections for the *Monte Carlo* method. All three sensitivity analyses are useful for quantifying the probability of successful implementation, increasing the production yield, and minimizing the cost of filter manufacturing.

A list of sensitivities of elementary functions is presented below:

$F(x)$	$S_x^{F(x)}$	$F(y(x), v(x))$	$S_x^{F(y(x),v(x))}$
x	1	$y(x)$	$S_{y(x)}^{F(y(x))} = S_y^{F(y)}$
cx	1	$y(x) \cdot v(x)$	$S_x^{y(x)} + S_x^{v(x)}$
c^x	$x \ \ln c$	$\lvert y(x)\rvert e^{jv(x)}$	$\text{Re}\ S_x^{\lvert y(x)\rvert e^{jv(x)}} + j\dfrac{\text{Im}\ S_x^{\lvert y(x)\rvert e^{jv(x)}}}{v(x)}$
$c + x$	$\dfrac{x}{c+x}$	$y(x) + v(x)$	$\dfrac{y(x)S_x^{y(x)} + v(x)S_x^{v(x)}}{y(x) + v(x)}$
$\dfrac{1}{x}$	-1	$\dfrac{y(x)}{v(x)}$	$S_x^{y(x)} - S_x^{v(x)}$
c	0	$\lvert y(x)\rvert$	$\text{Re}\ S_x^{y(x)}$
$x - c$	$\dfrac{x}{x-c}$	$y(x) - v(x)$	$\dfrac{y(x)S_x^{y(x)} - v(x)S_x^{v(x)}}{y(x) - v(x)}$
$\ln(x)$	$\dfrac{1}{\ln x}$	$e^{y(x)}$	$y(x)S_x^{y(x)}$
$\sqrt{x}$	$\frac{1}{2}$	$\ln(y(x))$	$\dfrac{1}{\ln(y(x))}S_x^{y(x)}$
$v^2(x)$	$2S_x^{v(x)}$	$y(v(x))$	$S_v^{y(v)}S_x^{v(x)}$ (4.44)

c is a constant, $y(x)$ and $v(x)$ are functions of x.

The *semirelative sensitivity* is defined as

$$S_{x_i}^F = x_i \frac{\partial F}{\partial x_i} = F\, S_{x_i}^F \tag{4.45}$$

4.6.2 Sensitivity of Second-Order Transfer Function

Let us consider a second-order transfer function of the form

$$H(s) = \frac{1}{s^2 + \dfrac{\omega_p}{Q_p}s + \omega_p^2} \tag{4.46}$$

Our goal is to find the sensitivities of the magnitude response, $M(\omega) = |H(j\omega)|$, with respect to the pole magnitude, ω_p, and the pole Q-factor, Q_p.

The squared magnitude response is

$$M^2(\omega) = \frac{1}{\left(-\omega^2 + \omega_p^2\right)^2 + \left(\dfrac{\omega_p}{Q_p}\right)^2 \omega^2} \tag{4.47}$$

The sensitivity of the magnitude response to the pole magnitude is

$$S_{\omega_p}^{M(\omega)}(\omega) = -\frac{2\left(1-\dfrac{\omega^2}{\omega_p^2}\right)+\dfrac{\omega^2}{\omega_p^2 Q_p^2}}{\left(1-\dfrac{\omega^2}{\omega_p^2}\right)^2+\dfrac{\omega^2}{\omega_p^2 Q_p^2}} \tag{4.48}$$

and the sensitivity to the pole Q-factor is

$$S_{Q_p}^{M(\omega)}(\omega) = \frac{\dfrac{\omega^2}{\omega_p^2 Q_p^2}}{\left(1-\dfrac{\omega^2}{\omega_p^2}\right)^2+\dfrac{\omega^2}{\omega_p^2 Q_p^2}} \tag{4.49}$$

To gain a better insight into the order of magnitudes of $S_{\omega_p}^{M(\omega)}$ and $S_{Q_p}^{M(\omega)}$, we plot them against the normalized frequency ω/ω_p in Figs. 4.15 and 4.16.

The extremes of $S_{\omega_p}^{M(\omega)}(\omega)$ are

$$\begin{aligned} S_{\omega_p}^{M(\omega)}(\omega)\Big|_{\min} &\approx -Q_p \qquad \text{for} \quad \frac{\omega}{\omega_p} \approx 1-\frac{1}{2Q_p} \text{ and } Q_p \gg 1 \\ S_{\omega_p}^{M(\omega)}(\omega)\Big|_{\max} &\approx Q_p \qquad \text{for} \quad \frac{\omega}{\omega_p} \approx 1+\frac{1}{2Q_p} \text{ and } Q_p \gg 1 \end{aligned} \tag{4.50}$$

while the extreme of $S_{Q_p}^{M(\omega)}$ is

$$S_{Q_p}^{M(\omega)}(\omega)\Big|_{\max} = 1 \tag{4.51}$$

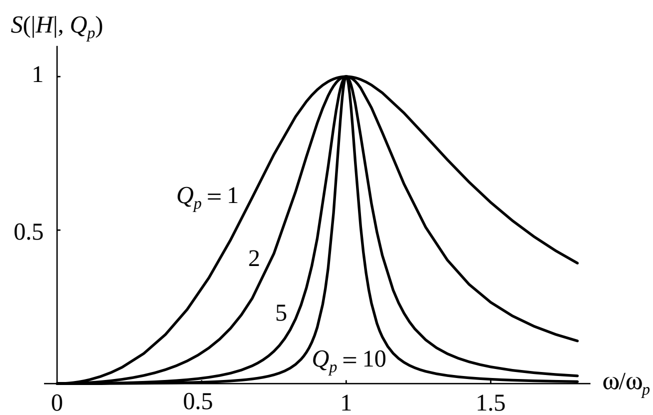

Figure 4.15 Sensitivities of the magnitude response to Q_p.

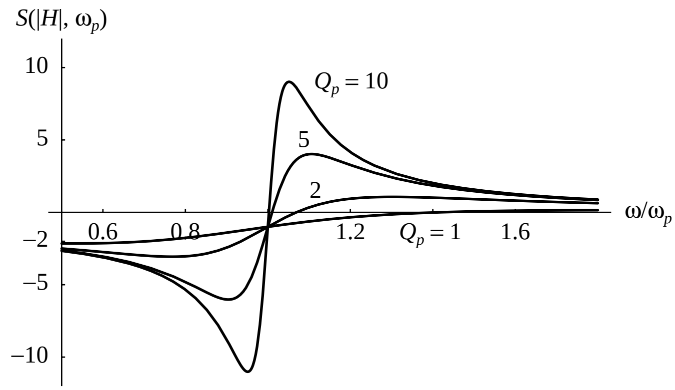

Figure 4.16 Sensitivities of the magnitude response to ω_p.

and is independent of Q_p. In the frequency range $0 \le \omega \le \omega_p$ the maximal value of the absolute sensitivity $\left|S_{\omega_p}^{M(\omega)}(\omega)\right|$ is Q_p times larger than $\left|S_{Q_p}^{M(\omega)}(\omega)\right|$:

$$\left|S_{\omega_p}^{M(\omega)}(\omega)\right|_{\max} \approx Q_p \left|S_{Q_p}^{M(\omega)}(\omega)\right|_{\max} \tag{4.52}$$

Summarizing the above analysis, we conclude that we can obtain the filter performance less sensitive to component deviations by decreasing Q_p and, also, by keeping the frequency $\omega_p\left(1 \pm 1/Q_p\right)$ away from the filter passband. Also, we have to pay more attention to the pole-magnitude sensitivity to element values than to the pole Q-factor sensitivity to component values.

The upper limit of the relative variation of the magnitude response can be expressed in the form

$$\left.\frac{\Delta M(\omega)}{M(\omega)}\right|_{\text{worst case}} = \sum_i \left|S_{\omega_p}^{M(\omega)} S_{x_i}^{\omega_p} \frac{\Delta x_i}{x_i}\right| + \sum_i \left|S_{Q_p}^{M(\omega)} S_{x_i}^{Q_p} \frac{\Delta x_i}{x_i}\right| \tag{4.53}$$

where x_i represents an element value of the chosen realization. The magnitude-response sensitivity is affected by

- the sensitivities $S_{\omega_p}^{M(\omega)}$, $S_{Q_p}^{M(\omega)}$ that depend on the transfer function—that is, on the approximation step;
- the sensitivities $S_{x_i}^{\omega_p}$, $S_{x_i}^{Q_p}$, determined by the chosen realization (circuit)—that is, by the realization step;
- the component tolerances $\dfrac{\Delta x_i}{x_i}$, imposed by the available technologies (the implementation step).

It can be shown that the gain deviation in dB, $\Delta G(\omega)$, is proportional to the magnitude response sensitivity:

$$\Delta G(\omega) \approx S_{x_i}^{G(\omega)} \frac{\Delta x_i}{x_i} = \frac{20}{\ln 10} S_{x_i}^{M(\omega)} \frac{\Delta x_i}{x_i} \approx 8.686\ S_{x_i}^{M(\omega)} \frac{\Delta x_i}{x_i} \tag{4.54}$$

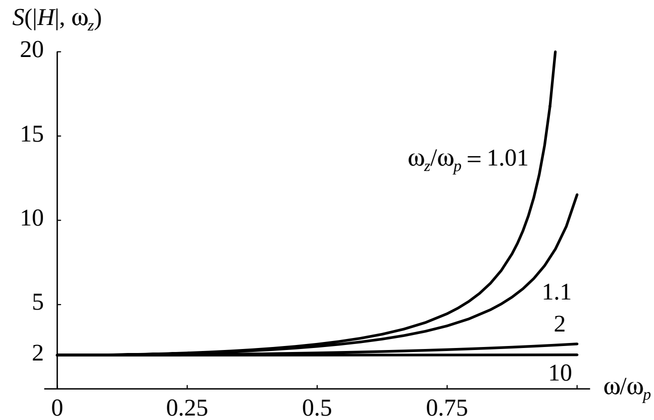

Figure 4.17 Sensitivities of the magnitude response to ω_z, for $\omega_z > \omega_p$.

where $G(\omega) = 20\log_{10} M(\omega)$, and x_i is an element value. Obviously, when optimizing a filter design it is irrelevant whether we optimize the gain sensitivity $S_{x_i}^{G(\omega)}$ or the magnitude response sensitivity $S_{x_i}^{M(\omega)}$.

Let us consider a second-order notch transfer function of the form

$$H(s) = \frac{s^2 + \omega_z^2}{s^2 + \dfrac{\omega_p}{Q_p}s + \omega_p^2} \tag{4.55}$$

The sensitivity of the magnitude response to the magnitude of the transfer function zero is

$$S_{\omega_z}^{M(\omega)}(\omega) = \frac{2}{1 - \dfrac{\omega^2}{\omega_p^2}\dfrac{\omega_p^2}{\omega_z^2}} \tag{4.56}$$

We plot the sensitivity $S_{\omega_z}^{M(\omega)}$ against the normalized frequency ω/ω_p in Fig. 4.17.

4.6.3 Sensitivity to Passive Components

Analog filter realizations consist of passive circuit elements, resistors, capacitors, and inductors. We define the *summed sensitivity* of a function $F(x_1, \ldots, x_n)$ to all elements of the same type $x_1, \ldots, x_n$ as

$$S_x^F = \sum_{i=1}^{n} S_{x_i}^F \tag{4.57}$$

where $S_{x_i}^F$ is defined by Eq. (4.38).

We mention without proof an important property that holds for all active RC filters with n_R resistors and n_C capacitors [28]:

$$\sum_{i=1}^{n_R} S_{R_i}^{Q_p} = \sum_{k=1}^{n_C} S_{C_k}^{Q_p} = 0 \tag{4.58}$$

$$\sum_{i=1}^{n_R} S_{R_i}^{\omega_p} = \sum_{k=1}^{n_C} S_{C_k}^{\omega_p} = -1 \tag{4.59}$$

where the summation is over all the resistors (capacitors).

In practice, the relative changes of elements of the same type can be $\frac{\Delta R_i}{R_i} = \frac{\Delta R}{R}$ and $\frac{\Delta C_k}{C_k} = \frac{\Delta C}{C}$. The relative variation of the magnitude response of an RC filter with $n = n_R + n_C$ passive elements becomes

$$\frac{\Delta M(\omega)}{M(\omega)} = S_{\omega_p}^{M(\omega)} \sum_{i=1}^{n} S_{x_i}^{\omega_p} \frac{\Delta x_i}{x_i} + S_{Q_p}^{M(\omega)} \sum_{i=1}^{n} S_{x_i}^{Q_p} \frac{\Delta x_i}{x_i} \tag{4.60}$$

$$\begin{aligned} \frac{\Delta M(\omega)}{M(\omega)} &= S_{\omega_p}^{M(\omega)} \left(\frac{\Delta R}{R} \sum_{i=1}^{n_R} S_{R_i}^{\omega_p} + \frac{\Delta C}{C} \sum_{k=1}^{n_C} S_{C_k}^{\omega_p} \right) \\ &+ S_{Q_p}^{M(\omega)} \left(\frac{\Delta R}{R} \sum_{i=1}^{n_R} S_{R_i}^{Q_p} + \frac{\Delta C}{C} \sum_{k=1}^{n_C} S_{C_k}^{Q_p} \right) \end{aligned} \tag{4.61}$$

According to Eq. (4.58) the second term is zero, and

$$\frac{\Delta M(\omega)}{M(\omega)} = S_{\omega_p}^{M(\omega)} \left(\frac{\Delta R}{R} \sum_{i=1}^{n_R} S_{R_i}^{\omega_p} + \frac{\Delta C}{C} \sum_{k=1}^{n_C} S_{C_k}^{\omega_p} \right) \tag{4.62}$$

For $\frac{\Delta R}{R} = -\frac{\Delta C}{C} = \frac{\Delta x}{x}$

$$\frac{\Delta M(\omega)}{M(\omega)} = S_{\omega_p}^{M(\omega)} \frac{\Delta x}{x} \left(\sum_{i=1}^{n_R} S_{R_i}^{\omega_p} - \sum_{k=1}^{n_C} S_{C_k}^{\omega_p} \right) \tag{4.63}$$

According to Eq. (4.59) the expression in parentheses is zero, and

$$\frac{\Delta M(\omega)}{M(\omega)} = 0 \tag{4.64}$$

which shows that the magnitude response of active RC filters can be made insensitive to the variation of passive components if $\frac{\Delta R_i}{R_i} = \frac{\Delta R}{R}$, $\frac{\Delta C_k}{C_k} = \frac{\Delta C}{C}$, and $\frac{\Delta R}{R} = -\frac{\Delta C}{C}$ hold.

The element values x, such as R_i or C_k, are spread about their nominal values x_0:

$$x = x_0(1 + \gamma_x) \tag{4.65}$$

where γ_x is a random number whose range is the tolerance x_t:

$$-\frac{x_t}{100} < \gamma_x < +\frac{x_t}{100} \tag{4.66}$$

The random number γ_x typically has a Gaussian distribution characterized by mean $\mu(\gamma_x)$ and standard deviation $\sigma(\gamma_x)$. In that case the tolerance corresponds to $3\sigma(\gamma_x)$—that is, $x_t = 300\sigma(\gamma_x)\%$.

The element value at temperature $T = T_0 + \Delta T$ is given by

$$x = x_0(1 + \alpha_x \Delta T) \tag{4.67}$$

where x_0 is the value at room temperature T_0 and α_x is the temperature coefficient of the component x. If a resistor has a temperature coefficient of 100 ppm/K (parts per million per kelvin), that is $\alpha_x = 100 \cdot 10^{-6}$ 1/K.

In a similar manner we can describe the effects of aging

$$x = x_0(1 + \beta_x \sqrt{t}) \tag{4.68}$$

where β_x is in ppm/yr. (parts per million per year) and t is time.

For small deviation from the nominal value the element value can be described as

$$x = x_0(1 + \gamma_x + \alpha_x \Delta T + \beta_x \sqrt{t}) \tag{4.69}$$

The relative deviation (variability) is

$$\frac{\Delta x}{x_0} = \gamma_x + \alpha_x \Delta T + \beta_x \sqrt{t} \tag{4.70}$$

the mean deviation is

$$\mu(\frac{\Delta x}{x_0}) \approx \mu(\gamma_x) + \mu(\alpha_x)\Delta T + \mu(\beta_x)\sqrt{t} \tag{4.71}$$

and the standard deviation per unit change in element value x is

$$\sigma\left(\frac{\Delta x}{x_0}\right) \approx \sqrt{(\sigma(\gamma_x))^2 + (\sigma(\alpha_x)\Delta T)^2 + (\sigma(\beta_x))^2 t} \tag{4.72}$$

4.6.4 Gain-Sensitivity Product (GSP)

Practical operational amplifiers (op amp) have finite and frequency-dependent gain that must be taken into account when designing active *RC* filters. Typically, the voltage gain, A, of an op amp can be approximated by

$$A(s) = A_0 \frac{1}{1 + \dfrac{s}{\omega_{3\text{dB}}}} = \frac{\omega_G}{s + \omega_{3\text{dB}}} \tag{4.73}$$

where A_0 is the DC gain ($A_0 \gg 1$), $\omega_{3\text{dB}}$ is the angular frequency at which $|A(j\omega_{3\text{dB}})| = A_0/\sqrt{2}$, and $\omega_G = A_0\omega_{3\text{dB}}$ is the gain–bandwidth product ($\omega_G \gg \omega_{3\text{dB}}$). Notice that at $s = j\omega_G$ we have

$$|A(j\omega_G)| \approx 1 \tag{4.74}$$

At DC and low frequencies we have

$$A(0) = A_0 = \frac{\omega_G}{\omega_{3\text{dB}}} \gg 1, \qquad \omega_G \gg \omega_{3\text{dB}} \tag{4.75}$$

At high frequencies we have

$$A(s) \approx A_0 \frac{\omega_{3\text{dB}}}{s} = \frac{\omega_G}{s}, \qquad \omega \gg \omega_{3\text{dB}} \tag{4.76}$$

In practice, we choose an op amp with a very large gain $|A(j\omega)|$ in the filter passband.

The finite op amp gain can produce considerable degradation of filter performances, so we have to study the effects of the op amp gain on the filter transfer function.

In order to quantify op amp imperfections, Moschytz [29] defines the *gain–sensitivity product* (GSP), Γ_A^F of a function F with respect to gain A:

$$\Gamma_A^F = A\ S_A^F \tag{4.77}$$

The main reason for using Γ_A^F instead of S_A^F is that S_A^F tends to be 0 for infinite values of A, and we cannot investigate the influence of A on filter performances. On the other hand, Γ_A^F is usually nonzero when $A \to \infty$, and the relative deviation of the function F is

$$\frac{\Delta F}{F} = \Gamma_A^F \frac{\Delta A}{A^2} \tag{4.78}$$

In this book we use the simplified figure-of-merit, I_A, recommended by reference 29, for the worst-case deviation of the magnitude response:

$$I_A = Q_p \Gamma_A^{\omega_p} + \frac{1}{2}\Gamma_A^{Q_p} \tag{4.79}$$

If a more complicated amplifier model than the single-pole model Eq. (4.73) is considered, then more complex relations for GSP and I_A have to be used as reported in reference 29. Fortunately, I_A is a good measure of performance for second-order active RC filters of practical importance.

For many second-order single op amp filters we have $\Gamma_A^{\omega_p} = 0$, and we can minimize the magnitude response sensitivity to A by optimizing $\Gamma_A^{Q_p}$ only.

4.7 ANALOG FILTER REALIZATIONS

After having accomplished the approximation step, the filter transfer function is known, and the designer must choose a realization—that is, an electric circuit. Analog filters can be classified on the basis of their constituent components as

- Passive RLC filters
- Operational amplifier RC filters (op amp active RC filters)
- Switched-capacitor (SC) filters
- Operational transconductance amplifier (OTA) filters

- Current-conveyor (CC) filters
- Microwave filters
- Electromechanical filters
- Crystal filters

Passive RLC filters consist of passive macrocomponents: resistors, capacitors, and inductors (coils and transformers). They do not require a power supply. The main drawbacks of passive filters are as follows: (a) They often exhibit a significant passband loss, and (b) the inductors cannot be miniaturized as required by modern applications. These filters are practical at frequencies up to a few hundred MHz. Passive *RLC* realizations are important in deriving realizations of some active filters, such as active *RC* filters, OTA filters, and current-conveyor filters.

Active RC filters, as well as *SC filters*, can be reduced in size and weight, especially when implemented as integrated circuits. Manufacturing of these filters can be automated with high production yield. They are made out of resistors, capacitors, switches and operational amplifiers. The disadvantages of active *RC* and SC filters are as follows: (a) They require a power supply, (b) the output signal can be distorted if the input signal is too large, and (c) an extra noise is generated in active devices. These filters operate over the frequency rage from 0.1 Hz up to 500 kHz.

Microwave filters are used for realization of filters above 300 MHz. They consist of microwave passive components, such as transmission line sections, coupled transmission lines, resonators, and cavities. They do not require a power supply.

Crystal filters are made of piezoelectric resonators. In filter applications, the Q-factor of a crystal resonator is very large, between 10^4 and 10^5, and the useful frequency range is from 10 kHz to 200 MHz. The relative bandwidth of a bandpass crystal filter using quartz resonators is very small, being less than 10^{-4}.

Electromechanical filters are made of mechanical resonators. The electrical signals are converted into mechanical vibrations and, after filtering, the resultant mechanical vibration is converted back to an electrical signal. The method of transforming electrical energy to mechanical energy and vice versa makes use of the magnetostrictive effect. Various mechanical solids can be made to vibrate in any one of many different modes such as longitudinal, extensional, torsional, and flexural. The Q-factor of the mechanical resonators can be 10^4. They can be used up to 200 MHz. The mechanical and piezoelectric resonators are based on the fact that at lower frequencies the exchange of the potential and kinetic energies in acoustical waves occurs much more efficiently than does a corresponding energy exchange in electromagnetic devices.

In this book we will focus on the realizations of passive *RLC* filters, op amp active *RC* filters, SC filters, OTA filters, and current-conveyor filters.

4.8 OP AMP ACTIVE *RC* FILTERS

Suppose that the filter transfer function has been determined and decomposed into second-order transfer functions. The filter can be realized as a cascade of second-order sections. Thus, the designer has to choose a realization for each biquadratic section. We adopt the classification of biquadratic realizations suggested by Moschytz and Horn [15 pp. 38–84].

The classification is based on the pole Q-factor:

- Low Q-factor realizations ($Q \leq 2$) exhibit, in general, no problems with tolerances, and there is no need for tuning. Therefore, the minimal number of passive components and operational amplifiers (one op amp per second-order section) has been chosen.
- Medium Q-factor realizations ($2 < Q \leq 20$) were selected on the basis of minimum gain-sensitivity product, simple tuning, minimal number of passive components, and only one operational amplifier.
- High Q-factor realizations ($20 < Q$) require two operational amplifiers; the sensitivities of these realizations are lower than the sensitivity of single-amplifier biquads.

We also present a general-purpose biquad with three amplifiers.

4.8.1 Low-Q-Factor Biquadratic Realizations

The most important feature of the presented low-Q-factor biquads is $\Gamma_A^{\omega_p} = 0$, which means that the pole magnitude is insensitive to the finite op amp gain.

Lowpass Low-Q-Factor Realization. The lowpass low-Q-factor realization is shown in Fig. 4.18 [15, pp. 38–39; 35].

The transfer function of the biquad from Fig. 4.18 is

$$H_{LP}(s) = \frac{V_4}{V_1} = K \frac{\omega_p^2}{s^2 + \dfrac{\omega_p}{Q_p}s + \omega_p^2} \tag{4.80}$$

The constant K is called the *gain constant* of the lowpass biquad and

$$K \leq 1 \tag{4.81}$$

The resistor and capacitor values are functions of the transfer function parameters K, Q_p, ω_p, the capacitance C_x, and an auxiliary quantity P. (P is resistnance ratio $P = R_3/R_1$). We set the value of C_x and P and compute the element values as

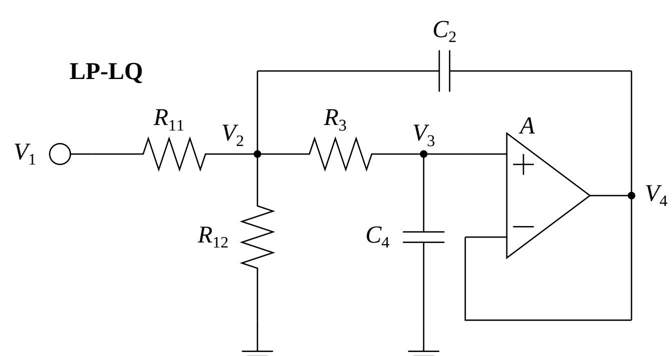

Figure 4.18 Lowpass low-Q-factor op amp biquad.

$$\boxed{\begin{aligned} R_1 &= \frac{1}{Q_p \omega_p C_x (1+P)} \\ R_{11} = R_{11}(K, Q_p, \omega_p, C_x, P) &= \frac{R_1}{K} \\ R_{12} = R_{12}(K, Q_p, \omega_p, C_x, P) &= \frac{R_1}{1-K} \\ C_2 = C_2(K, Q_p, \omega_p, C_x, P) &= Q_p^2 C_x \frac{(1+P)^2}{P} \\ R_3 = R_3(K, Q_p, \omega_p, C_x, P) &= P R_1 \\ C_4 = C_4(K, Q_p, \omega_p, C_x, P) &= C_x \end{aligned}} \tag{4.82}$$

Notice that P is the ratio of two resistances, $P = R_3/R_1$; these resistances affect the pole magnitude, $\omega_p = 1/(R_1 R_3 C_2 C_4)$. In practice, we prefer to choose P from the range $0.1 < P < 10$.

The intermediate quantity, R_1, is indented in (4.82).

If we can choose $K = 1$, then R_{12} becomes infinite and the resistor R_{12} degenerates to the open circuit (disappears from the schematic).

The quantity P can be used for optimizing an element value, or the element-value spread ratio. For example, we can vary P until C_2 gets a value from a prescribed set of values.

Highpass Low-Q-Factor Realization. The highpass low-Q-factor realization is shown in Fig. 4.19 [15, pp. 44–45; 35].

The transfer function of the biquad from Fig. 4.19 is

$$H_{HP}(s) = \frac{V_4}{V_1} = K \frac{s^2}{s^2 + \dfrac{\omega_p}{Q_p} s + \omega_p^2} \tag{4.83}$$

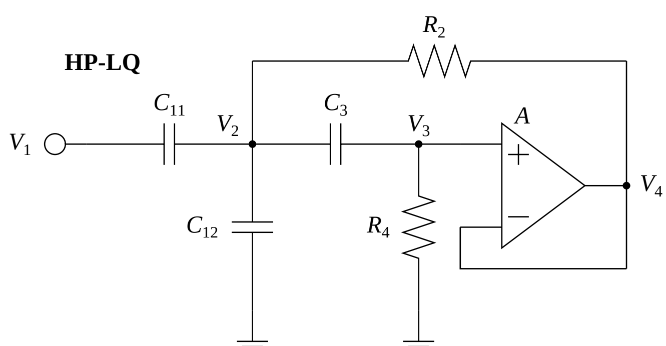

Figure 4.19 Highpass low-Q-factor op amp biquad.

The constant K is the gain constant of the highpass biquad and

$$K \leq 1 \tag{4.84}$$

The resistor and capacitor values are functions of the transfer function parameters K, Q_p, and ω_p, the capacitance C_x, and an auxiliary quantity P. (P is the resistance ratio $P = R_4/R_2$). We set the value of C_x and P and compute the element values as

$$\boxed{\begin{aligned} C_1 &= C_x \\ C_{11} &= C_{11}(K, Q_p, \omega_p, C_x, P) = K C_x \\ C_{12} &= C_{12}(K, Q_p, \omega_p, C_x, P) = C_x - C_{11} \\ C_3 &= C_3(K, Q_p, \omega_p, C_x, P) = C_x \frac{P - 2Q_p^2 - \sqrt{P^2 - 4PQ_p^2}}{2Q_p^2} \\ R_2 &= R_2(K, Q_p, \omega_p, C_x, P) = \frac{1}{Q_p \omega_p (C_1 + C_3)} \\ R_4 &= R_4(K, Q_p, \omega_p, C_x, P) = P R_2 \end{aligned}} \tag{4.85}$$

A good choice for P is $0.1 < P < 10$.

Bandpass Low-Q-Factor Realization. The bandpass low-Q-factor realization is shown in Fig. 4.20 [15, pp. 40–41; 36].

The transfer function of the biquad from Fig. 4.20 is

$$H_{BP}(s) = \frac{V_4}{V_1} = K \frac{\dfrac{\omega_p}{Q_p} s}{s^2 + \dfrac{\omega_p}{Q_p} s + \omega_p^2} \tag{4.86}$$

The constant K is the gain constant of the bandpass biquad.

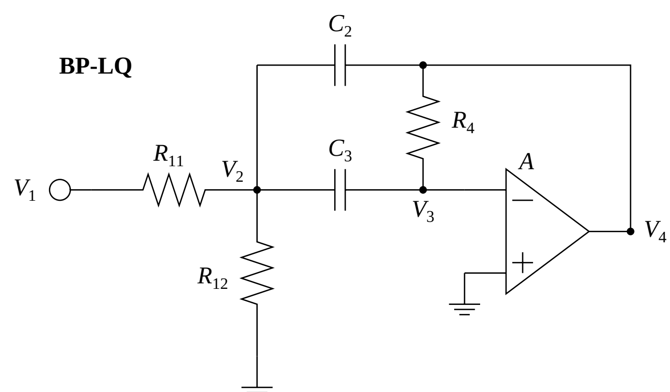

Figure 4.20 Bandpass low-Q-factor op amp biquad.

The resistor and capacitor values are functions of the transfer function parameters K, Q_p, and ω_p, the capacitance C_x, and an auxiliary quantity P. (P is resistance ratio $P = R_4/R_1$). We set the value of C_x and P and compute the element values as

$$\boxed{\begin{aligned} C_2 &= C_2(K, Q_p, \omega_p, C_x, P) = C_x \\ C_3 &= C_3(K, Q_p, \omega_p, C_x, P) = C_x \frac{P - 2Q_p^2 - \sqrt{P^2 - 4PQ_p^2}}{2Q_p^2} \\ R_1 &= \frac{1}{Q_p\omega_p(C_2 + C_3)} \\ R_4 &= R_4(K, Q_p, \omega_p, C_x, P) = PR_1 \\ R_{11} &= R_{11}(K, Q_p, \omega_p, C_x, P) = \frac{C_3R_4}{K(C_2 + C_3)} \\ R_{12} &= R_{12}(K, Q_p, \omega_p, C_x, P) = \frac{C_3R_1R_4}{C_3R_4 - R_1K(C_2 + C_3)} \end{aligned}} \tag{4.87}$$

Notice that P is the ratio of two resistances, $P = R_4/R_1$; these resistances affect the pole magnitude, $\omega_p = 1/(R_1R_4C_2C_3)$, where $R_1 = 1/(1/R_{11} + 1/R_{12})$. Obviously,

$$P \geq 4Q_p^2 \tag{4.88}$$

For $P = 4Q_p^2$ the capacitors have the same value $C_2 = C_3$.

For $P = 4Q_p^2$, and $K = 2Q_p^2$, the resistance R_{12} becomes infinite; that is, R_{12} disappears from the schematic.

Bandreject Low-Q-Factor Realization. The bandreject low-Q-factor realization is shown in Fig. 4.21 [15, pp. 50–51; 37].

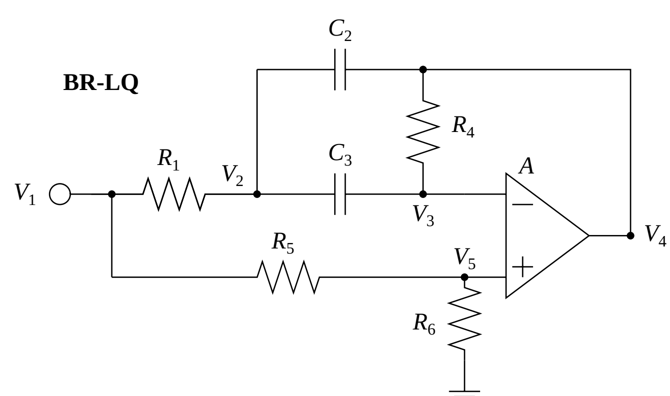

Figure 4.21 Bandreject low-Q-factor op amp biquad.

The transfer function of the biquad from Fig. 4.21 is

$$H_{BR}(s) = \frac{V_4}{V_1} = K \frac{s^2 + \omega_p^2}{s^2 + \dfrac{\omega_p}{Q_p} s + \omega_p^2} \tag{4.89}$$

The constant K is the gain constant of the bandreject biquad, and

$$K < 1 \tag{4.90}$$

The resistor and capacitor values are functions of the transfer function parameters K, Q_p, and ω_p, the capacitance C_x, and the resistance R_x. We set the value of C_x and R_x and compute the element values as

$$\boxed{\begin{aligned}
C_2 &= C_2(K, Q_p, \omega_p, C_x, R_x) = C_x \\
P &= \frac{4}{\left(\dfrac{1}{K} - 1\right)\left(2 - \left(\dfrac{1}{K} - 1\right) Q_p^2\right)} \\
C_3 &= C_3(K, Q_p, \omega_p, C_x, P, R_x) = C_x \frac{P - 2Q_p^2 - \sqrt{P^2 - 4PQ_p^2}}{2Q_p^2} \\
R_1 &= R_1(K, Q_p, \omega_p, C_x, P, R_x) = \frac{1}{Q_p \omega_p (C_2 + C_3)} \\
R_4 &= R_4(K, Q_p, \omega_p, C_x, P, R_x) = P R_1 \\
R_5 &= R_5(K, Q_p, \omega_p, C_x, P, R_x) = R_x \left(\frac{1}{K} - 1\right) \\
R_6 &= R_6(K, Q_p, \omega_p, C_x, P, R_x) = R_x
\end{aligned}} \tag{4.91}$$

It should be noticed that K and Q_p must satisfy

$$Q_p < \frac{1}{\sqrt{\dfrac{1}{K} - 1}} \tag{4.92}$$

and

$$\frac{1}{1 + \dfrac{1}{Q_p^2}} < K < 1 \tag{4.93}$$

to ensure positive element values.

If we choose

$$K = \frac{1}{1 + \dfrac{1}{2Q_p^2}} \tag{4.94}$$

we obtain

$$P \geq 4Q_p^2 \tag{4.95}$$

and the capacitor values become equal, $C_2 = C_3 = C_x$.

Notice that P is the ratio of two resistances, $P = R_4/R_1$; these resistances affect the pole magnitude, $\omega_p = 1/(R_1 R_4 C_2 C_3)$.

Allpass Low-Q-Factor Realization. The allpass low-Q-factor realization is shown in Fig. 4.22 [15, pp. 48–49; 37].

The transfer function of the biquad from Fig. 4.22 is

$$H_{AP}(s) = \frac{V_4}{V_1} = K \frac{s^2 - \dfrac{\omega_p}{Q_p} s + \omega_p^2}{s^2 + \dfrac{\omega_p}{Q_p} s + \omega_p^2} \tag{4.96}$$

The constant K is the gain constant of the allpass biquad.

The resistor and capacitor values are functions of the transfer function parameters K, Q_p, and ω_p, the capacitance C_x, and the resistance R_x. We set the value of C_x and

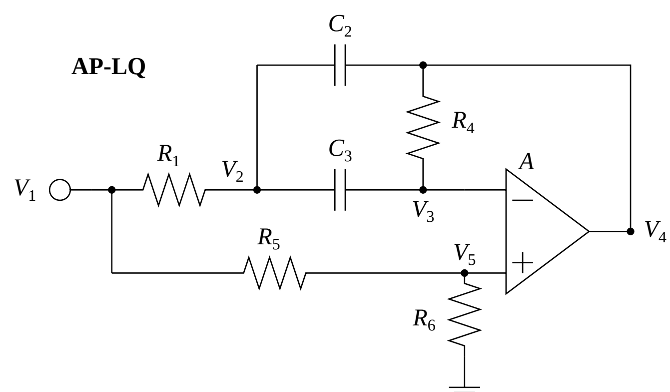

Figure 4.22 Allpass low-Q-factor op amp biquad.

R_x and compute the element values as

$$\boxed{\begin{aligned}
C_2 &= C_2(K, Q_p, \omega_p, C_x, R_x) = C_x \\
P &= \frac{4}{\left(\frac{1}{K} - 1\right)\left(2 - \left(\frac{1}{K} - 1\right) Q_p^2\right)} \\
C_3 &= C_3(K, Q_p, \omega_p, C_x, P, R_x) = C_x \frac{P - 2Q_p^2 - \sqrt{P^2 - 4PQ_p^2}}{2Q_p^2} \\
R_1 &= R_1(K, Q_p, \omega_p, C_x, P, R_x) = \frac{1}{Q_p \omega_p (C_2 + C_3)} \\
R_4 &= R_4(K, Q_p, \omega_p, C_x, P, R_x) = P R_1 \\
R_5 &= R_5(K, Q_p, \omega_p, C_x, P, R_x) = R_x \left(\frac{1}{K} - 1\right) \\
R_6 &= R_6(K, Q_p, \omega_p, C_x, P, R_x) = R_x
\end{aligned}} \tag{4.97}$$

It should be noticed that K must satisfy

$$\frac{1}{1 + \frac{2}{Q_p^2}} < K < 1 \tag{4.98}$$

to ensure positive element values.

If we choose

$$K = \frac{1}{1 + \frac{1}{Q_p^2}} \tag{4.99}$$

the capacitor values become equal, $C_2 = C_3 = C_x$.

4.8.2 Medium-Q-Factor Biquadratic Realizations

The most important feature of the presented medium-Q-factor biquads, except for notch biquads, is $\Gamma_A^{\omega_p} = 0$, which means that the pole magnitude is insensitive to the finite op amp gain.

Lowpass Medium-Q-Factor Realization. The lowpass medium-Q-factor realization is shown in Fig. 4.23[15, pp. 52–53; 35].

The transfer function of the biquad from Fig. 4.23 is

$$H_{LP}(s) = \frac{V_4}{V_1} = K \frac{\omega_p^2}{s^2 + \frac{\omega_p}{Q_p} s + \omega_p^2} \tag{4.100}$$

The constant K is the gain constant of the lowpass biquad.

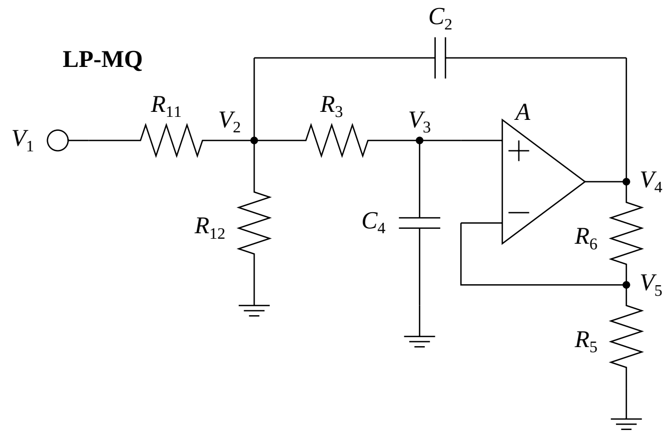

Figure 4.23 Lowpass medium-Q-factor op amp biquad.

The resistor and capacitor values are functions of the transfer function parameters K, Q_p, and ω_p, the capacitances C_{2x} and C_{4x}, the resistance R_x, and an auxiliary quantity P. (P is resistance ratio $P = R_3/R_1$). We set the value of C_{2x}, C_{4x}, R_x, and P and compute the element values as

$$
\boxed{
\begin{aligned}
C_2 &= C_2(K, Q_p, \omega_p, C_{2x}, C_{4x}, P, R_x) = C_{2x} \\
C_4 &= C_4(K, Q_p, \omega_p, C_{2x}, C_{4x}, P, R_x) = C_{4x} \\
R_1 &= \frac{1}{\omega_p \sqrt{C_{2x} C_{4x} P}} \\
R_3 &= R_3(K, Q_p, \omega_p, C_{2x}, C_{4x}, P, R_x) = P R_1 \\
R_5 &= R_5(K, Q_p, \omega_p, C_{2x}, C_{4x}, P, R_x) = R_x \\
R_6 &= R_6(K, Q_p, \omega_p, C_{2x}, C_{4x}, P, R_x) = R_x \left(\frac{C_4(1 + P)}{C_2} - \frac{\sqrt{P \frac{C_4}{C_2}}}{Q_p} \right) \\
K_0 &= 1 + \frac{R_6}{R_x} \\
R_{11} &= R_{11}(K, Q_p, \omega_p, C_{2x}, C_{4x}, P, R_x) = \frac{R_1 K_0}{K} \\
R_{12} &= R_{12}(K, Q_p, \omega_p, C_{2x}, C_{4x}, P, R_x) = \frac{R_1 K_0}{K_0 - K}
\end{aligned}
}
\qquad (4.101)
$$

The gain constant K can be larger than 1.

Highpass Medium-Q-Factor Realization. The highpass medium-Q-factor realization is shown in Fig. 4.24 [15, pp. 58–59; 35].

The transfer function of the biquad from Fig. 4.24 is

$$H_{HP}(s) = \frac{V_4}{V_1} = K \frac{s^2}{s^2 + \dfrac{\omega_p}{Q_p} s + \omega_p^2} \tag{4.102}$$

The constant K is the gain constant of the highpass biquad.

The resistor and capacitor values are functions of the transfer function parameters K, Q_p, and ω_p, the capacitances C_{1x} and C_{3x}, the resistance R_x, and an auxiliary quantity P. (P is resistance ratio $P = R_4/R_2$). We set the value of C_{1x}, C_{3x}, R_x, and P and compute the element values as

$$\boxed{\begin{aligned}
C_1 &= C_{1x} \\
C_3 &= C_3(K, Q_p, \omega_p, C_{1x}, C_{3x}, P, R_x) = C_{3x} \\
R_2 &= R_2(K, Q_p, \omega_p, C_{1x}, C_{3x}, P, R_x) = \frac{1}{\omega_p \sqrt{C_{1x} C_{3x} P}} \\
R_4 &= R_4(K, Q_p, \omega_p, C_{1x}, C_{3x}, P, R_x) = P R_2 \\
R_5 &= R_5(K, Q_p, \omega_p, C_{1x}, C_{3x}, P, R_x) = R_x \\
R_6 &= R_6(K, Q_p, \omega_p, C_{1x}, C_{3x}, P, R_x) = R_x \left(\frac{1 + \dfrac{C_1}{C_3}}{P} - \frac{\sqrt{\dfrac{C_1}{P C_3}}}{Q_p} \right) \\
K_0 &= 1 + \frac{R_6}{R_x} \\
C_{11} &= C_{11}(K, Q_p, \omega_p, C_{1x}, C_{3x}, P, R_x) = \frac{C_1 K}{K_0} \\
C_{12} &= C_{12}(K, Q_p, \omega_p, C_{1x}, C_{3x}, P, R_x) = C_1 - C_{11}
\end{aligned}} \tag{4.103}$$

The gain constant K can be larger than 1.

Bandpass Medium-Q-Factor Realization. The bandpass medium-Q-factor realization is shown in Fig. 4.25 [15, pp. 54–55; 38].

The transfer function of the biquad from Fig. 4.25 is

$$H_{BP}(s) = \frac{V_4}{V_1} = K \frac{\dfrac{\omega_p}{Q_p} s}{s^2 + \dfrac{\omega_p}{Q_p} s + \omega_p^2} \tag{4.104}$$

The constant K is the gain constant of the bandpass biquad.

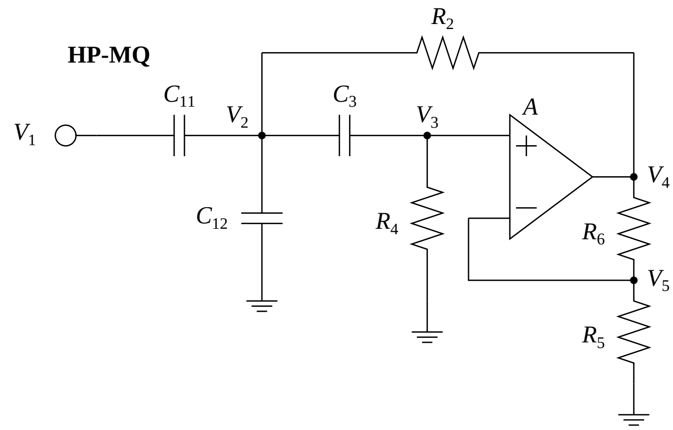

Figure 4.24 Highpass medium-Q-factor op amp biquad.

The resistor and capacitor values are functions of the transfer function parameters K, Q_p, and ω_p, the capacitances C_{2x} and C_{3x}, the resistance R_x, and an auxiliary quantity P. (P is resistance ratio $P = R_4/R_1$). We set the value of C_{2x}, C_{3x}, R_x, and P and compute the element values as

$$
\boxed{
\begin{aligned}
C_2 &= C_2(K, Q_p, \omega_p, C_{2x}, C_{3x}, P, R_x) = C_{2x} \\
C_3 &= C_3(K, Q_p, \omega_p, C_{2x}, C_{3x}, P, R_x) = C_{3x} \\
R_1 &= \frac{1}{\omega_p \sqrt{C_{2x} C_{3x} P}} \\
R_4 &= R_4(K, Q_p, \omega_p, C_{2x}, C_{3x}, P, R_x) = P R_1 \\
R_6 &= R_6(K, Q_p, \omega_p, C_{2x}, C_{3x}, P, R_x) = R_x \\
R_5 &= R_5(K, Q_p, \omega_p, C_{2x}, C_{3x}, P, R_x) = R_x \left(\frac{1 + \dfrac{C_2}{C_3}}{P} - \frac{\sqrt{\dfrac{C_2}{P C_3}}}{Q_p} \right) \\
K_0 &= Q_p \left(1 + \frac{R_5}{R_x} \right) \sqrt{\frac{P C_3}{C_2}} \\
R_{11} &= R_{11}(K, Q_p, \omega_p, C_{2x}, C_{3x}, P, R_x) = \frac{R_1 K_0}{K} \\
R_{12} &= R_{12}(K, Q_p, \omega_p, C_{2x}, C_{3x}, P, R_x) = \frac{R_1 K_0}{K_0 - K}
\end{aligned}
}
\tag{4.105}
$$

The gain constant K can be larger than 1.

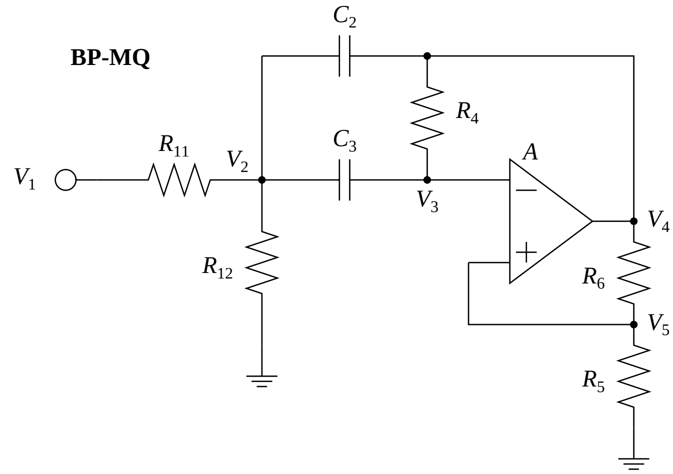

Figure 4.25 Bandpass medium-Q-factor op amp biquad.

Bandreject Medium-Q-Factor Realization. The bandreject medium-Q-factor realization is shown in Fig. 4.26 [15, pp. 62–63; 29].

The transfer function of the biquad from Fig. 4.26 is

$$H_{BR}(s) = \frac{V_4}{V_1} = \frac{s^2 + \omega_p^2}{s^2 + \dfrac{\omega_p}{Q_p}s + \omega_p^2} \tag{4.106}$$

The resistor and capacitor values are functions of the transfer function parameters Q_p and ω_p, the capacitances C_{2x} and C_{3x}, the resistance R_x, and an auxiliary quantity

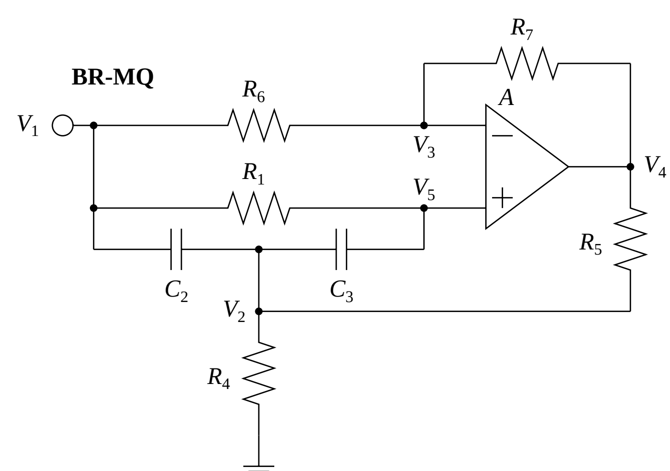

Figure 4.26 Bandreject medium-Q-factor op amp biquad.

P. We set the value of C_{2x}, C_{3x}, R_x, and P and compute the element values as

$$\boxed{\begin{aligned}
C_2 &= C_2(Q_p, \omega_p, C_{2x}, C_{3x}, P, R_x) = C_{2x} \\
C_3 &= C_3(Q_p, \omega_p, C_{2x}, C_{3x}, P, R_x) = C_{3x} \\
R_1 &= R_1(Q_p, \omega_p, C_{2x}, C_{3x}, P, R_x) = \frac{1}{\omega_p\sqrt{C_{2x}C_{3x}P}} \\
R_p &= PR_1 \\
R_6 &= R_6(Q_p, \omega_p, C_{2x}, C_{3x}, P, R_x) = R_x \\
R_7 &= R_7(Q_p, \omega_p, C_{2x}, C_{3x}, P, R_x) = R_x P\left(1 + \frac{C_2}{C_3}\right) \\
a &= 1 - \frac{\sqrt{\frac{PC_2}{C_3}}}{Q_p\left(1 + \frac{R_7}{R_x}\right)} \\
R_5 &= R_5(Q_p, \omega_p, C_{2x}, C_{3x}, P, R_x) = \frac{R_p}{a} \\
R_4 &= R_4(Q_p, \omega_p, C_{2x}, C_{3x}, P, R_x) = \frac{R_p}{1-a}
\end{aligned}} \tag{4.107}$$

Notice that the gain constant is 1.

Allpass Medium-Q-Factor Realization. The allpass medium-Q-factor realization is shown in Fig. 4.27 [15, pp. 60–61; 29].

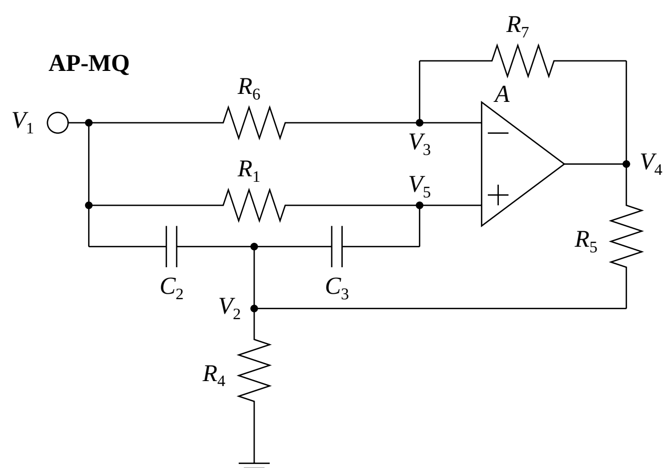

Figure 4.27 Allpass medium-Q-factor op amp biquad.

The transfer function of the biquad from Fig. 4.27 is

$$H_{AP}(s) = \frac{V_4}{V_1} = \frac{s^2 - \dfrac{\omega_p}{Q_p}s + \omega_p^2}{s^2 + \dfrac{\omega_p}{Q_p}s + \omega_p^2} \tag{4.108}$$

The resistor and capacitor values are functions of the transfer function parameters Q_p and ω_p, the capacitances C_{2x} and C_{3x}, the resistance R_x, and an auxiliary quantity P. We set the value of C_{2x}, C_{3x}, R_x, and P and compute the element values as

$$\boxed{\begin{aligned}
C_2 &= C_2(Q_p, \omega_p, C_{2x}, C_{3x}, P, R_x) = C_{2x} \\
C_3 &= C_3(Q_p, \omega_p, C_{2x}, C_{3x}, P, R_x) = C_{3x} \\
R_1 &= R_1(Q_p, \omega_p, C_{2x}, C_{3x}, P, R_x) = \frac{1}{\omega_p \sqrt{C_{2x} C_{3x} P}} \\
R_p &= P R_1 \\
R_6 &= R_6(Q_p, \omega_p, C_{2x}, C_{3x}, P, R_x) = R_x \\
R_7 &= R_7(Q_p, \omega_p, C_{2x}, C_{3x}, P, R_x) = R_x \left(P\left(1 + \frac{C_2}{C_3}\right) + \frac{\sqrt{P\dfrac{C_2}{C_3}}}{Q_p} \right) \\
a &= 1 - \frac{2\sqrt{\dfrac{P C_2}{C_3}}}{Q_p\left(1 + \dfrac{R_7}{R_x}\right)} \\
R_5 &= R_5(Q_p, \omega_p, C_{2x}, C_{3x}, P, R_x) = \frac{R_p}{a} \\
R_4 &= R_4(Q_p, \omega_p, C_{2x}, C_{3x}, P, R_x) = \frac{R_p}{1 - a}
\end{aligned}} \tag{4.109}$$

Notice that the gain constant is 1.

Lowpass Notch Medium-Q-Factor Realization. The lowpass notch medium-Q-factor realization is shown in Fig. 4.28 [15, pp. 64–65; 39].

The transfer function of the biquad from Fig. 4.28 is

$$H_{LPN}(s) = \frac{V_4}{V_1} = K\frac{s^2 + \omega_z^2}{s^2 + \dfrac{\omega_p}{Q_p}s + \omega_p^2}, \qquad \omega_z > \omega_p \tag{4.110}$$

The constant K is the gain constant of the notch biquad.

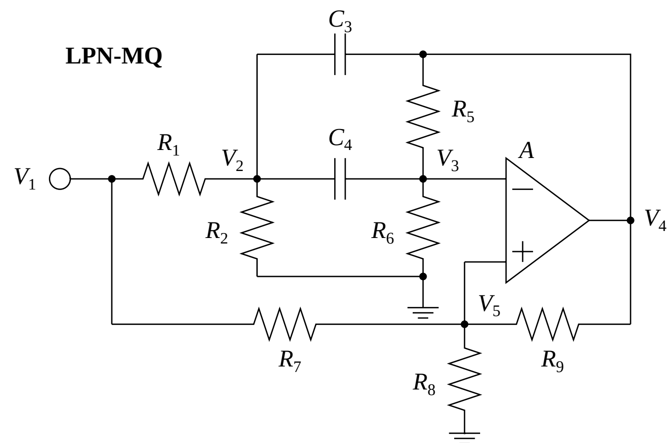

Figure 4.28 Lowpass notch medium-Q-factor op amp biquad.

The resistor and capacitor values are functions of the transfer function parameters K, Q_p, ω_p, and ω_z, the capacitances C_{3x} and C_{4x}, the resistance R_x, and an auxiliary quantity P. We set the value of C_{3x}, C_{4x}, R_x, and P and compute the element values as

$$
\boxed{
\begin{aligned}
C_3 &= C_3(K, Q_p, \omega_p, \omega_z, P, C_{3x}, C_{4x}, R_x) = C_{3x} \\
C_4 &= C_4(K, Q_p, \omega_p, \omega_z, P, C_{3x}, C_{4x}, R_x) = C_{4x} \\
G &= \frac{C_3\omega_p}{2PQ_p}\left(\sqrt{1 + 4Q_p^2 P\left(1 + \frac{C_4}{C_3}\right)} - 1\right) \\
K_0 &= \frac{1 + P}{1 + \left(1 + \frac{C_4}{C_3}\right)\omega_z^2 \frac{C_3^2}{G^2}} \\
R_1 &= R_1(K, Q_p, \omega_p, \omega_z, P, C_{3x}, C_{4x}, R_x) = \frac{K_0}{KG} \\
R_2 &= R_2(K, Q_p, \omega_p, \omega_z, P, C_{3x}, C_{4x}, R_x) = \frac{K_0}{G\,(K_0 - K)} \\
R_6 &= R_6(K, Q_p, \omega_p, \omega_z, P, C_{3x}, C_{4x}, R_x) = \frac{G(1 + P)}{C_3C_4(\omega_z^2 - \omega_p^2)} \\
R_5 &= R_5(K, Q_p, \omega_p, \omega_z, P, C_{3x}, C_{4x}, R_x) = \frac{1}{\frac{C_3C_4\omega_p^2}{G} + \frac{P}{R_6}} \\
R_7 &= R_7(K, Q_p, \omega_p, \omega_z, P, C_{3x}, C_{4x}, R_x) = \frac{PR_x}{K} \\
R_8 &= R_8(K, Q_p, \omega_p, \omega_z, P, C_{3x}, C_{4x}, R_x) = \frac{PR_x}{1 - K} \\
R_9 &= R_9(K, Q_p, \omega_p, \omega_z, P, C_{3x}, C_{4x}, R_x) = R_x
\end{aligned}
}
\tag{4.111}
$$

The gain constant K can be greater than 1.

Highpass Notch Medium-Q Factor Realization. The highpass notch medium-Q-factor realization is shown in Fig. 4.29 [15, pp. 64–65; 39].

The transfer function of the biquad from Fig. 4.29 is

$$H_{HPN}(s) = \frac{V_4}{V_1} = K\frac{s^2 + \omega_z^2}{s^2 + \dfrac{\omega_p}{Q_p}s + \omega_p^2}, \qquad \omega_z < \omega_p \tag{4.112}$$

The constant K is the gain constant of the notch biquad.

The resistor and capacitor values are functions of the transfer function parameters K, Q_p, ω_p, and ω_z, the capacitances C_{3x}, and C_{4x}, the resistance R_x, and an auxiliary quantity P. We set the value of C_{3x}, C_{4x}, R_x, and P and compute the element values as

$$\boxed{\begin{aligned}
C_3 &= C_3(K, Q_p, \omega_p, \omega_z, P, C_{3x}, C_{4x}, R_x) = C_{3x}\\
C_4 &= C_4(K, Q_p, \omega_p, \omega_z, P, C_{3x}, C_{4x}, R_x) = C_{4x}\\
G &= \frac{C_3\omega_p}{2PQ_p}\left(\sqrt{1 + 4Q_p^2P\left(1 + \frac{C_4}{C_3}\right)} - 1\right)\\
K_0 &= \frac{1 + P}{1 + \left(1 + \dfrac{C_4}{C_3}\right)\omega_z^2\dfrac{C_3^2}{G^2}}\\
R_1 &= R_1(K, Q_p, \omega_p, \omega_z, P, C_{3x}, C_{4x}, R_x) = \frac{K_0}{KG}\\
R_2 &= R_2(K, Q_p, \omega_p, \omega_z, P, C_{3x}, C_{4x}, R_x) = \frac{K_0}{G\,(K_0 - K)}\\
R_6 &= R_6(K, Q_p, \omega_p, \omega_z, P, C_{3x}, C_{4x}, R_x) = \frac{G(1 + P)(1 - \frac{1}{K})}{C_3C_4(\omega_z^2 - \omega_p^2)}\\
R_5 &= R_5(K, Q_p, \omega_p, \omega_z, P, C_{3x}, C_{4x}, R_x) = \frac{1}{\dfrac{C_3C_4\omega_p^2}{G} + \dfrac{P}{R_6}}\\
R_7 &= R_7(K, Q_p, \omega_p, \omega_z, P, C_{3x}, C_{4x}, R_x) = \frac{PR_x}{K}\\
R_8 &= R_8(K, Q_p, \omega_p, \omega_z, P, C_{3x}, C_{4x}, R_x) = \frac{PR_x}{1 - K}\\
R_9 &= R_9(K, Q_p, \omega_p, \omega_z, P, C_{3x}, C_{4x}, R_x) = R_x
\end{aligned}} \tag{4.113}$$

The gain constant K can be greater than 1.

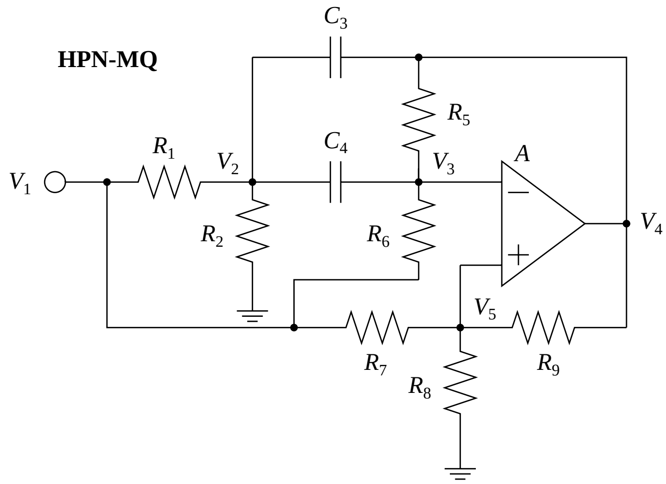

Figure 4.29 Highpass notch medium-Q-factor op amp biquad.

4.8.3 High-Q-Factor Biquadratic Realizations

High-Q-factor realizations require two operational amplifiers. The sensitivities of these realizations are lower than the sensitivity of single-amplifier biquads. The gain constant is equal to 1 for all presented realizations.

Lowpass High-Q-Factor Realization. The lowpass high-Q-factor realization is shown in Fig. 4.30 [15, pp. 68–69; 40].

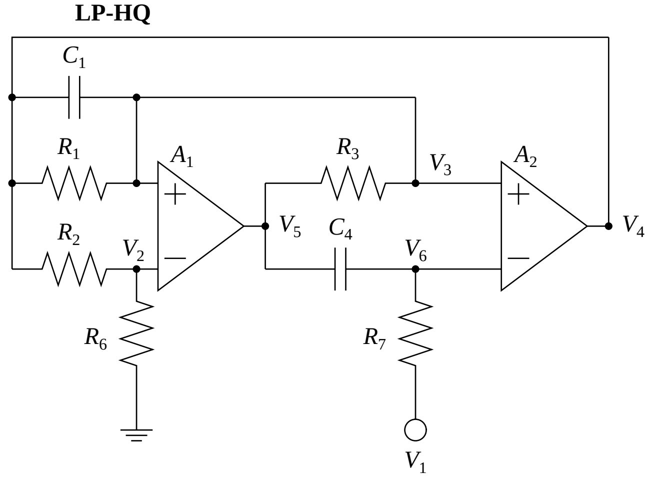

Figure 4.30 Lowpass high-Q-factor op amp biquad.

The transfer function of the biquad from Fig. 4.30 is

$$H_{LP}(s) = \frac{V_5}{V_1} = \frac{\omega_p^2}{s^2 + \dfrac{\omega_p}{Q_p}s + \omega_p^2} \tag{4.114}$$

The resistor and capacitor values are functions of the transfer function parameters Q_p and ω_p, the capacitance C_x, and the resistance R_x. We set the value of C_x, and R_x and compute the element values as

$$\boxed{\begin{aligned} C_1 &= C_1(Q_p, \omega_p, C_x, R_x) = C_x \\ C_4 &= C_4(Q_p, \omega_p, C_x, R_x) = C_x \\ R_0 &= \frac{1}{\omega_p C_x} \\ R_2 &= R_2(Q_p, \omega_p, C_x, R_x) = R_x \\ R_3 &= R_3(Q_p, \omega_p, C_x, R_x) = R_x \\ R_6 &= R_6(Q_p, \omega_p, C_x, R_x) = R_x \\ R_1 &= R_1(Q_p, \omega_p, C_x, R_x) = Q_p R_0 \\ R_7 &= R_7(Q_p, \omega_p, C_x, R_x) = \frac{R_0^2}{R_x} \end{aligned}} \tag{4.115}$$

Highpass High-Q-Factor Realization. The highpass high-Q-factor realization is shown in Fig. 4.31 [15, pp. 72–73; 40, 41].

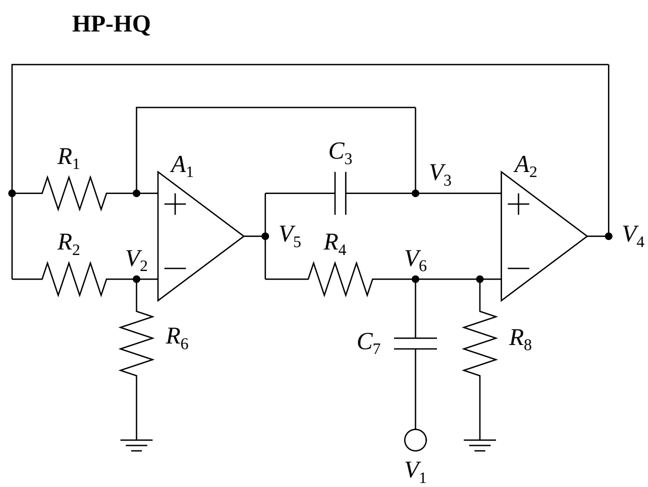

Figure 4.31 Highpass high-Q-factor op amp biquad.

The transfer function of the biquad from Fig. 4.31 is

$$H_{HP}(s) = \frac{V_4}{V_1} = \frac{s^2}{s^2 + \dfrac{\omega_p}{Q_p}s + \omega_p^2} \tag{4.116}$$

The resistor and capacitor values are functions of the transfer function parameters Q_p and ω_p, the capacitance C_x, and the resistance R_x. We set the value of C_x and R_x and compute the element values as

$$\boxed{\begin{aligned} C_3 &= C_3(Q_p, \omega_p, C_x, R_x) = C_x \\ C_7 &= C_7(Q_p, \omega_p, C_x, R_x) = C_x \\ R_0 &= \frac{1}{\omega_p C_x} \\ R_1 &= R_1(Q_p, \omega_p, C_x, R_x) = R_x \\ R_2 &= R_2(Q_p, \omega_p, C_x, R_x) = R_x \\ R_6 &= R_6(Q_p, \omega_p, C_x, R_x) = R_x \\ R_8 &= R_8(Q_p, \omega_p, C_x, R_x) = Q_p R_0 \\ R_4 &= R_4(Q_p, \omega_p, C_x, R_x) = \frac{R_0^2}{R_x} \end{aligned}} \tag{4.117}$$

Bandpass High-Q-Factor Realization. The bandpass high-Q-factor realization is shown in Fig. 4.32 [15, pp. 70–71; 40, 41].

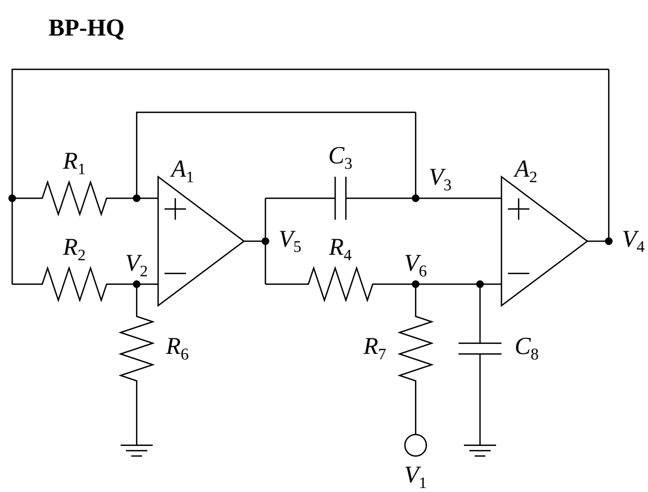

Figure 4.32 Bandpass high-Q-factor op amp biquad.

The transfer function of the biquad from Fig. 4.32 is

$$H_{BP}(s) = \frac{V_4}{V_1} = \frac{\dfrac{\omega_p}{Q_p}s}{s^2 + \dfrac{\omega_p}{Q_p}s + \omega_p^2} \tag{4.118}$$

The resistor and capacitor values are functions of the transfer function parameters Q_p and ω_p, the capacitance C_x, and the resistance R_x. We set the value of C_x and R_x and compute the element values as

$$\boxed{\begin{aligned}
C_3 &= C_3(Q_p, \omega_p, C_x, R_x) = C_x \\
C_8 &= C_8(Q_p, \omega_p, C_x, R_x) = C_x \\
R_0 &= \frac{1}{\omega_p C_x} \\
R_1 &= R_1(Q_p, \omega_p, C_x, R_x) = R_x \\
R_2 &= R_2(Q_p, \omega_p, C_x, R_x) = R_x \\
R_6 &= R_6(Q_p, \omega_p, C_x, R_x) = R_x \\
R_7 &= R_7(Q_p, \omega_p, C_x, R_x) = Q_p R_0 \\
R_4 &= R_4(Q_p, \omega_p, C_x, R_x) = \frac{R_0^2}{R_x}
\end{aligned}} \tag{4.119}$$

Bandreject High-Q-Factor Realization. The bandreject high-Q-factor realization is shown in Fig. 4.33 [15, pp. 76–77; 41].

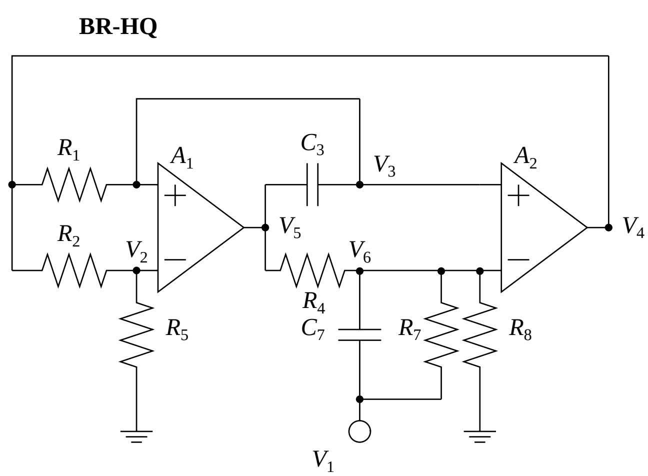

Figure 4.33 Bandreject high-Q-factor op amp biquad.

The transfer function of the biquad from Fig. 4.33 is

$$H_{BR}(s) = \frac{V_4}{V_1} = \frac{s^2 + \omega_p^2}{s^2 + \dfrac{\omega_p}{Q_p}s + \omega_p^2} \tag{4.120}$$

The resistor and capacitor values are functions of the transfer function parameters Q_p and ω_p, the capacitance C_x, and the resistance R_x. We set the value of C_x and R_x and compute the element values as

$$\boxed{\begin{aligned} C_3 &= C_3(Q_p, \omega_p, C_x, R_x) = C_x \\ C_7 &= C_7(Q_p, \omega_p, C_x, R_x) = C_x \\ R_0 &= \frac{1}{\omega_p C_x} \\ R_1 &= R_1(Q_p, \omega_p, C_x, R_x) = R_x \\ R_2 &= R_2(Q_p, \omega_p, C_x, R_x) = R_x \\ R_5 &= R_5(Q_p, \omega_p, C_x, R_x) = R_x \\ R_7 &= R_7(Q_p, \omega_p, C_x, R_x) = 2Q_p R_0 \\ R_8 &= R_8(Q_p, \omega_p, C_x, R_x) = 2Q_p R_0 \\ R_4 &= R_4(Q_p, \omega_p, C_x, R_x) = \frac{R_0^2}{R_x} \end{aligned}} \tag{4.121}$$

Allpass High-Q-Factor Realization. The allpass high-Q-factor realization is known in Fig. 4.34 [15, pp. 74–75; 40, 41].

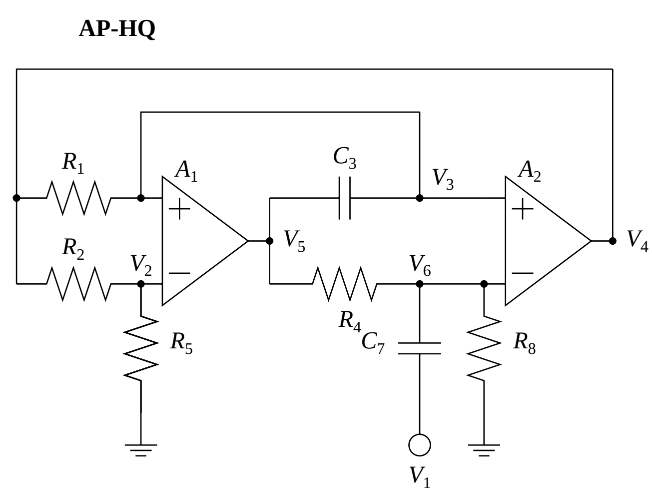

Figure 4.34 Allpass high-Q-factor op amp biquad.

The transfer function of the biquad from Fig. 4.34 is

$$H_{AP}(s) = \frac{V_4}{V_1} = \frac{s^2 - \dfrac{\omega_p}{Q_p}s + \omega_p^2}{s^2 + \dfrac{\omega_p}{Q_p}s + \omega_p^2} \tag{4.122}$$

The resistor and capacitor values are functions of the transfer function parameters Q_p and ω_p, the capacitance C_x, and the resistance R_x. We set the value of C_x and R_x and compute the element values as

$$\boxed{\begin{aligned}
C_3 &= C_3(Q_p, \omega_p, C_x, R_x) = C_x \\
C_7 &= C_7(Q_p, \omega_p, C_x, R_x) = C_x \\
R_0 &= \frac{1}{\omega_p C_x} \\
R_1 &= R_1(Q_p, \omega_p, C_x, R_x) = R_x \\
R_2 &= R_2(Q_p, \omega_p, C_x, R_x) = R_x \\
R_5 &= R_5(Q_p, \omega_p, C_x, R_x) = R_x \\
R_8 &= R_8(Q_p, \omega_p, C_x, R_x) = Q_p R_0 \\
R_4 &= R_4(Q_p, \omega_p, C_x, R_x) = \frac{R_0^2}{R_x}
\end{aligned}} \tag{4.123}$$

Lowpass Notch High-Q-Factor Realization. The lowpass notch high-Q-factor realization is shown in Fig. 4.35 [15, pp. 78–79; 41].

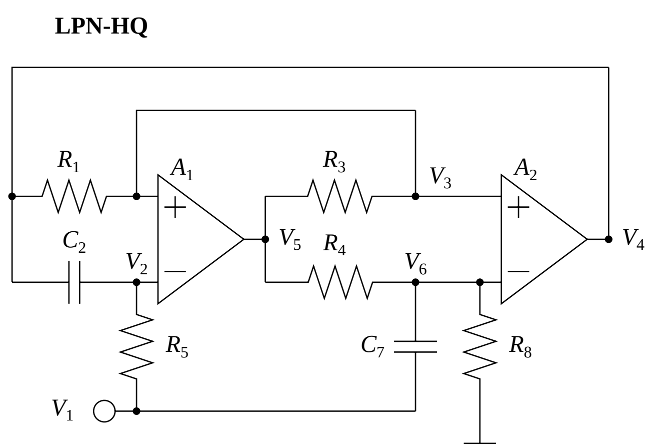

Figure 4.35 Lowpass notch high-Q-factor op amp biquad.

The transfer function of the biquad from Fig. 4.35 is

$$H_{LPN}(s) = \frac{V_5}{V_1} = \frac{s^2 + \omega_z^2}{s^2 + \dfrac{\omega_p}{Q_p}s + \omega_p^2}, \qquad \omega_z > \omega_p \tag{4.124}$$

The resistor and capacitor values are functions of the transfer function parameters Q_p, ω_p, and ω_z, the capacitance C_x, and the resistance R_x. We set the value of C_x and R_x and compute the element values as

$$\boxed{\begin{aligned} C_2 &= C_2(Q_p, \omega_p, \omega_z, C_x, R_x) = C_x \\ C_7 &= C_7(Q_p, \omega_p, \omega_z, C_x, R_x) = C_x \\ R_0 &= \frac{1}{\omega_p C_x} \\ R_1 &= R_1(Q_p, \omega_p, \omega_z, C_x, R_x) = R_x \\ R_3 &= R_3(Q_p, \omega_p, \omega_z, C_x, R_x) = R_x \\ R_8 &= R_8(Q_p, \omega_p, \omega_z, C_x, R_x) = Q_p R_0 \\ R_4 &= R_4(Q_p, \omega_p, \omega_z, C_x, R_x) = R_8\left(\frac{\omega_z^2}{\omega_p^2} - 1\right) \\ R_5 &= R_5(Q_p, \omega_p, \omega_z, C_x, R_x) = \frac{R_0^2}{R_4} \end{aligned}} \tag{4.125}$$

Highpass Notch High-Q-Factor Realization. The highpass notch high-Q-factor realization is shown in Fig. 4.36 [15, pp. 78–79; 41].

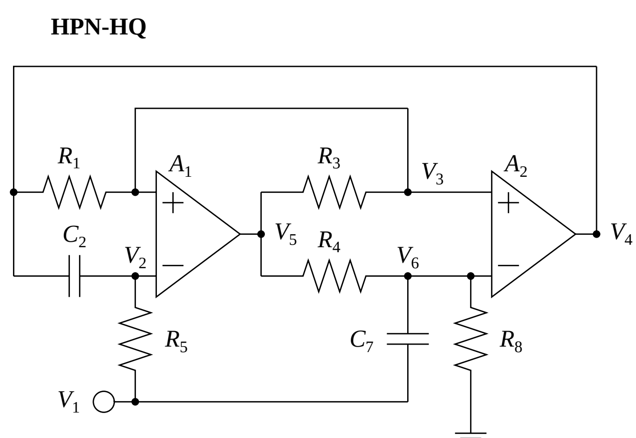

Figure 4.36 Highpass notch high-Q-factor op amp biquad.

The transfer function of the biquad from Fig. 4.36 is

$$H_{HPN}(s) = \frac{V_4}{V_1} = \frac{s^2 + \omega_z^2}{s^2 + \dfrac{\omega_p}{Q_p}s + \omega_p^2} \quad \omega_z < \omega_p \tag{4.126}$$

The resistor and capacitor values are functions of the transfer function parameters Q_p, ω_p, and ω_z, the capacitance C_x, and the resistance R_x. We set the value of C_x and R_x and compute the element values as

$$\boxed{\begin{aligned} C_2 &= C_2(Q_p, \omega_p, \omega_z, C_x, R_x) = C_x \\ C_7 &= C_7(Q_p, \omega_p, \omega_z, C_x, R_x) = C_x \\ R_0 &= \frac{1}{\omega_p C_x} \\ R_1 &= R_1(Q_p, \omega_p, \omega_z, C_x, R_x) = R_x \\ R_3 &= R_3(Q_p, \omega_p, \omega_z, C_x, R_x) = R_x \\ R_8 &= R_8(Q_p, \omega_p, \omega_z, C_x, R_x) = Q_p R_0 \\ R_4 &= R_4(Q_p, \omega_p, \omega_z, C_x, R_x) = R_8\left(1 - \frac{\omega_z^2}{\omega_p^2}\right) \\ R_5 &= R_5(Q_p, \omega_p, \omega_z, C_x, R_x) = \frac{R_0^2}{R_4} \end{aligned}} \tag{4.127}$$

General-Purpose Realization. The general-purpose realization, sometimes called the *KHN filter* (Kerwin–Huelsman–Newcomb), is shown in Fig. 4.37 [15, pp. 80–81; 42].

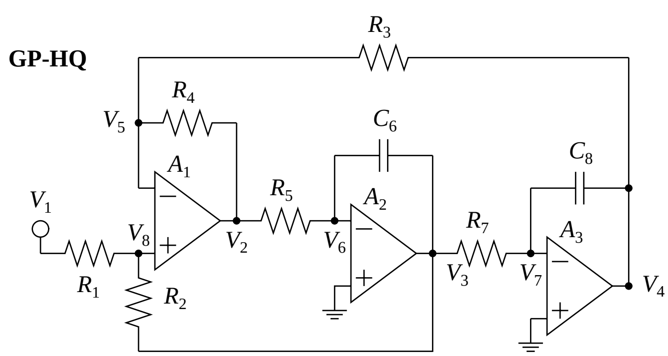

Figure 4.37 General-purpose op amp biquad.

The realizable transfer functions of the biquad from Fig. 4.37 are

$$
\begin{aligned}
H_{LP}(s) &= \frac{V_4}{V_1} = K_{LP}\frac{\omega_p^2}{s^2 + \dfrac{\omega_p}{Q_p}s + \omega_p^2} \\
H_{HP}(s) &= \frac{V_2}{V_1} = K_{HP}\frac{s^2}{s^2 + \dfrac{\omega_p}{Q_p}s + \omega_p^2} \\
H_{BP}(s) &= \frac{V_3}{V_1} = K_{BP}\frac{\dfrac{\omega_p}{Q_p}s}{s^2 + \dfrac{\omega_p}{Q_p}s + \omega_p^2}
\end{aligned}
\tag{4.128}
$$

The resistor and capacitor values are functions of the transfer function parameters Q_p and ω_p, the capacitance C_x, and the resistance R_x. We set the value of C_x and R_x and compute the element values as

$$
\boxed{
\begin{aligned}
C_6 &= C_6(Q_p, \omega_p, C_x, R_x) = C_x \\
C_8 &= C_8(Q_p, \omega_p, C_x, R_x) = C_x \\
R_0 &= \frac{1}{\omega_p C_x} \\
R_1 &= R_1(Q_p, \omega_p, C_x, R_x) = R_x \\
R_3 &= R_3(Q_p, \omega_p, C_x, R_x) = R_x \\
R_5 &= R_5(Q_p, \omega_p, C_x, R_x) = R_x \\
R_7 &= R_7(Q_p, \omega_p, C_x, R_x) = R_x \\
R_4 &= R_4(Q_p, \omega_p, C_x, R_x) = \frac{R_x^3}{R_0^2} \\
R_2 &= R_2(Q_p, \omega_p, C_x, R_x) = R_x\left(\frac{Q_p\left(1 + \dfrac{R_4}{R_x}\right)}{\sqrt{\dfrac{R_4}{R_x}}} - 1\right)
\end{aligned}
}
\tag{4.129}
$$

The gain constants are

$$
\begin{aligned}
K_{LP} &= \frac{R_2(R_3 + R_4)}{R_4(R_1 + R_2)} \\
K_{HP} &= \frac{R_2(R_3 + R_4)}{R_3(R_1 + R_2)} \\
K_{BP} &= -\frac{R_2}{R_1}
\end{aligned}
$$

If a gain constant K is given, we can find R_x as a function of Q_p, ω_p, C_x and that gain constant K. Next, we proceed according to Eq. (4.129).

4.9 SWITCHED-CAPACITOR (SC) FILTERS

Switched-capacitor (SC) filters implemented as integrated circuits offer high accuracy, relatively low price, straightforward design, and small number of external components. The benefits of SC filters are as follows:

- no attenuation in passband;
- possible gain in passband;
- realization of all basic break transfer-function types (lowpass, highpass, bandpass, bandreject, allpass, notch and bump) with one universal circuit;
- fully inductorless implementation;
- no external capacitors;
- small number of external resistors, or no external resistors in fully integrated implementation;
- high-input impedance;
- low-output impedance;
- small size and weight;
- easy tuning;
- low-frequency operation (as low as 0.1 Hz)
- simple design equations;
- short time-to-market interval.

The drawbacks of SC filters are as follows:

- noise associated with active devices;
- limited dynamic range to about 80 dB;
- small amount of clock-frequency signal feedthrough appears at the output;
- high-frequency operation limited to approximately 200 kHz.

The operation of SC filters is based on the fact that resistance can be simulated by using switches and capacitors. By definition, the resistance of a resistor is the ratio of the voltage across the resistor to the current through the resistor. When a grounded capacitor C is repeatedly switched at the clock frequency f_{CLK}, between a constant voltage source V_s and the ground, the average current that flows into the capacitor is equal to the capacitor's charge $q = V_s C$, multiplied by the clock frequency f_{CLK}:

$$I_{\text{average}} = V_s C f_{CLK} \tag{4.130}$$

We define the equivalent resistance of the switched capacitor as a ratio of the source voltage to the average current. It follows that the resistance is inversely proportional to the product of the capacitance, C, and the clock frequency, f_{CLK}:

$$R = \frac{V_s}{I_{\text{average}}} = \frac{1}{C f_{CLK}} \tag{4.131}$$

Typically, the capacitor tolerances of integrated SC filters can be more than 30%, which is too large for filter applications. However, for a unity capacitance ratio, $r_c = C_1/C_2 \approx 1$, a 0.1% accuracy of r_c can be achieved. In fact, for filter applications, a high

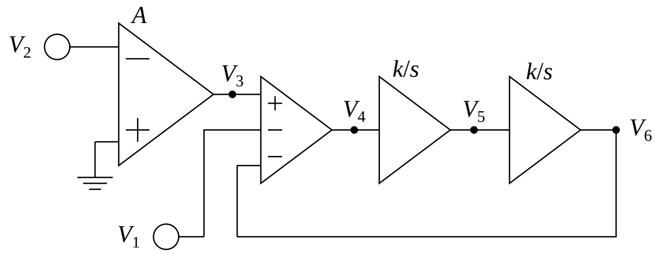

Figure 4.38 Universal integrated SC circuit with internal feedback.

accuracy of pole magnitudes should be obtained. The pole magnitude of an SC filter, ω_p, is proportional to the capacitance ratio $r_c = C_1/C_2$ and the clock frequency f_{CLK}:

$$\omega_p = \frac{1}{R_1 C_2} = \frac{1}{\dfrac{1}{C_1 f_{CLK}} C_2} = \frac{C_1}{C_2} f_{CLK} = r_c f_{CLK} \tag{4.132}$$

where R_1 is the equivalent resistance of the switched capacitor C_1. Therefore, SC integrated filters can accurately realize poles because they have a very good matching of integrated capacitors, r_c, and an accurate clock frequency, f_{CLK}.

A universal integrated SC filter is based on the state-variable structure. This structure consists of four active components: an uncommitted operational amplifier, a unique three-input summing stage, and two integrators. The transfer function of the integrators is $\frac{k}{s}$. One of the inputs of the summing stage can be connected through an internal switch to the output of the second integrator (Fig. 4.38) or to the ground (Fig. 4.39).

The integration constant k is proportional to the clock frequency f_{CLK}:

$$k = \frac{2\pi}{P} f_{CLK}, \qquad P = 50 \quad \text{or} \quad P = 100 \tag{4.133}$$

where P can be set by an external control signal.

The grounded mode of operation (Fig. 4.39) has a double pole at infinity. Finite poles can be realized by feeding the signal from the output of the second integrator

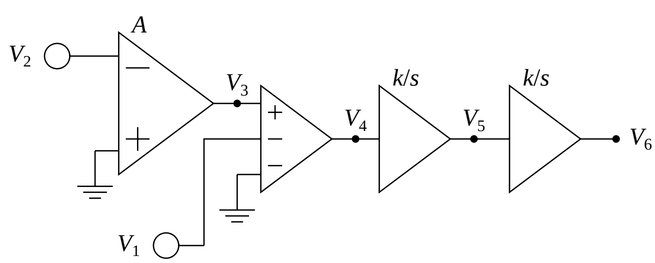

Figure 4.39 Universal integrated SC circuit with grounded summing stage input.

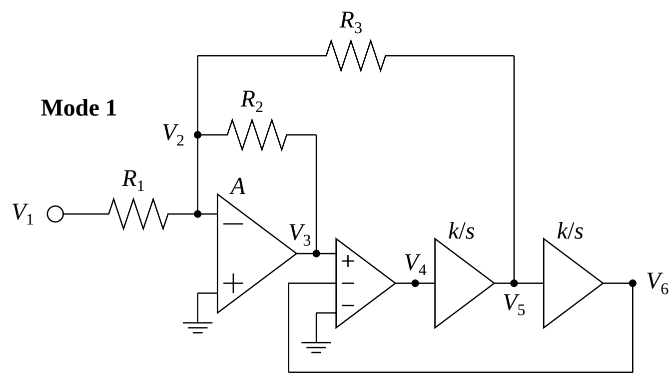

Figure 4.40 Mode 1 SC biquad.

(through a resistor) to the operational amplifier. In this case, the operational amplifier acts as a summing amplifier.

The feedback mode of operation (Fig. 4.38) realizes finite poles, $\omega_p = \frac{2\pi}{P} f_{CLK}$. Pole magnitudes different from $\frac{2\pi}{P} f_{CLK}$ can be realized by additional feedback from the output of the second integrator (through a resistor) to the operational amplifier.

The universal integrated SC circuit, along with additional resistors, operates in several modes that are classified according to the type of realized transfer function or feedback connection (Figs. 4.40–4.55) [43]–[45].

Modes of operation 1, 2, 3, 4, and 5 are used for realizations of second-order transfer functions Eqs. (134)–(140). Modes of operation 6 and 7 are used for realizations of first-order transfer functions, Eqs. (134) and (141).

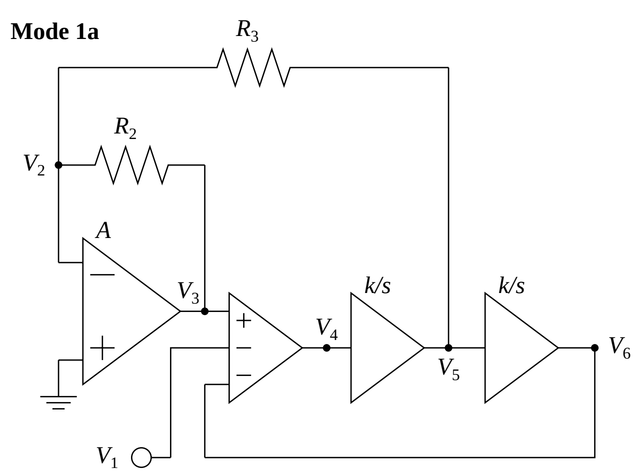

Figure 4.41 Mode 1a SC biquad.

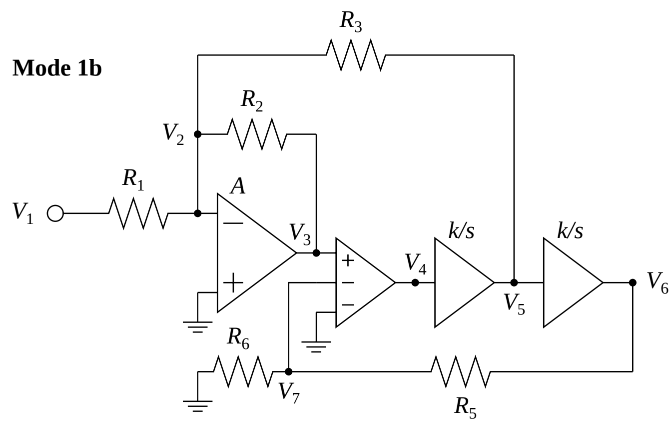

Figure 4.42 Mode 1b SC biquad.

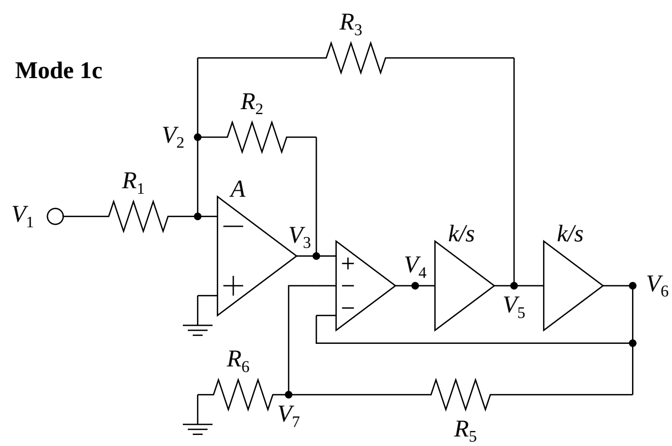

Figure 4.43 Mode 1c SC biquad.

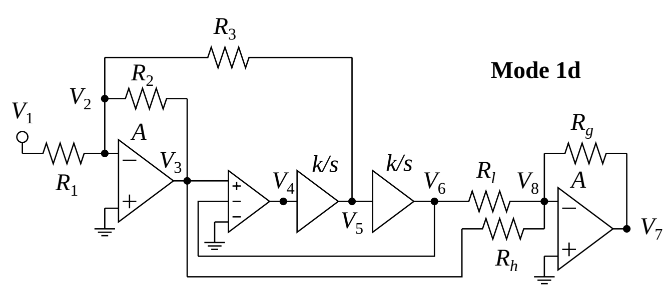

Figure 4.44 Mode 1d SC biquad.

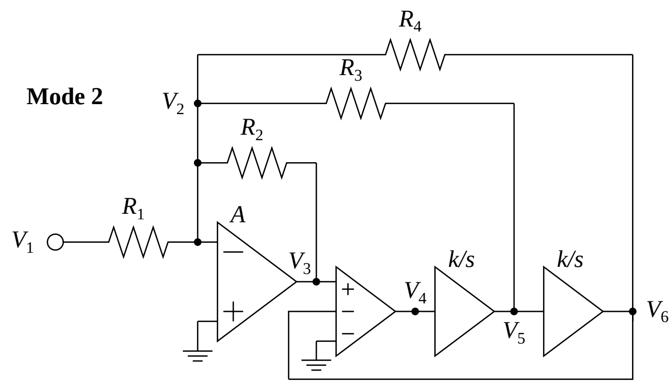

Figure 4.45 Mode 2 SC biquad.

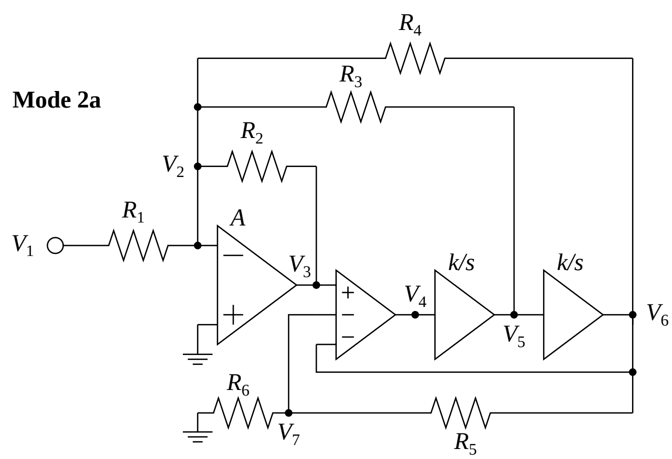

Figure 4.46 Mode 2a SC biquad.

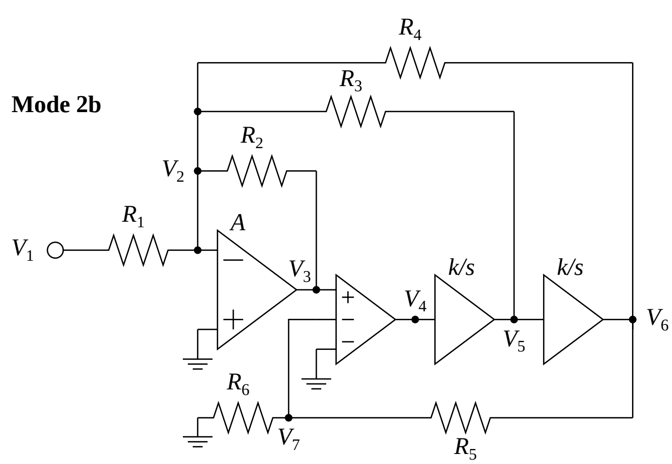

Figure 4.47 Mode 2b SC biquad.

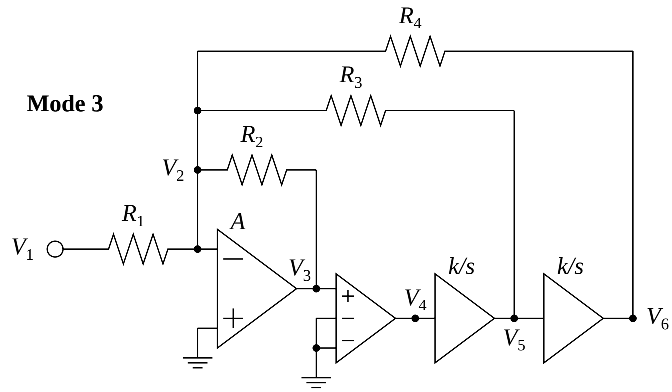

Figure 4.48 Mode 3 SC biquad.

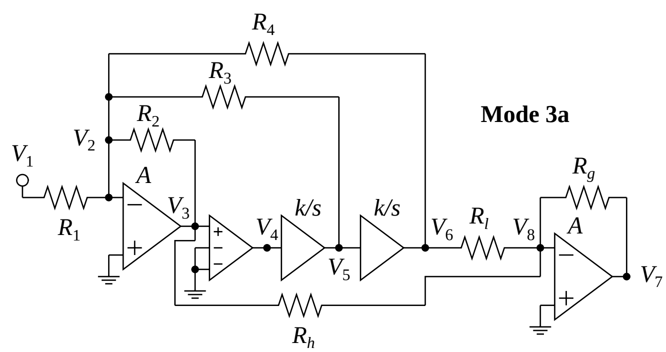

Figure 4.49 Mode 3a SC biquad.

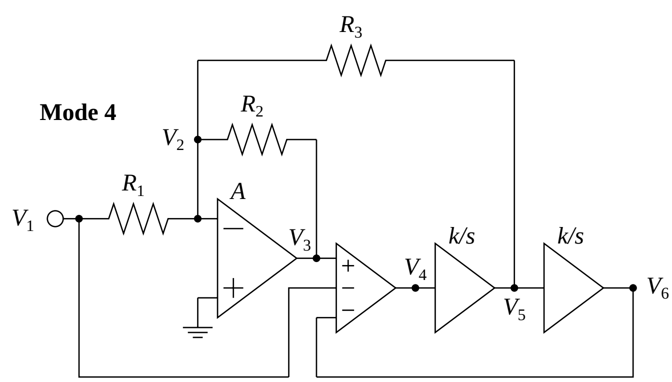

Figure 4.50 Mode 4 SC biquad.

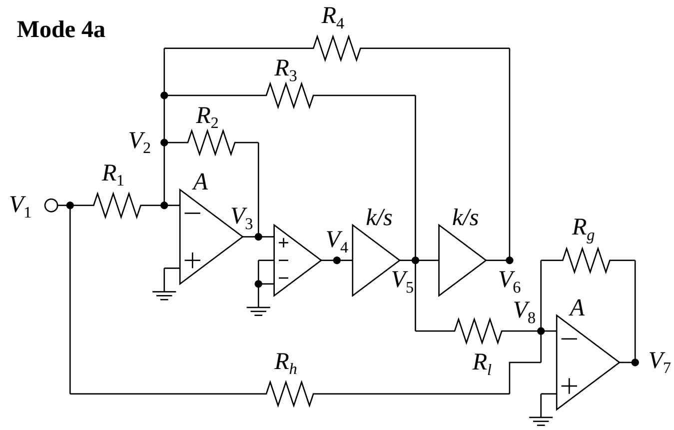

Figure 4.51 Mode 4a SC biquad.

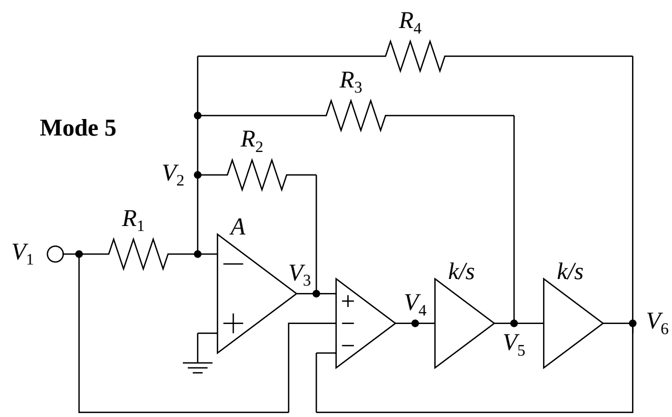

Figure 4.52 Mode 5 SC biquad.

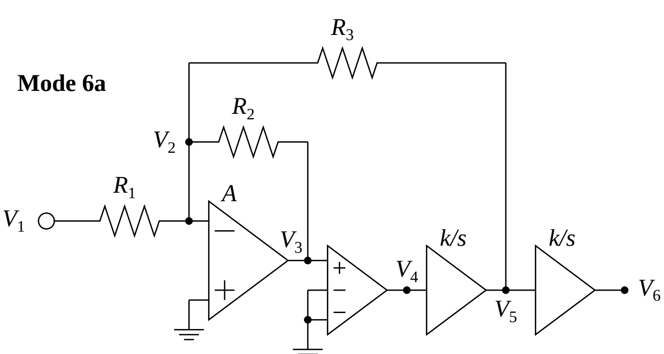

Figure 4.53 Mode 6a SC first-order realization.

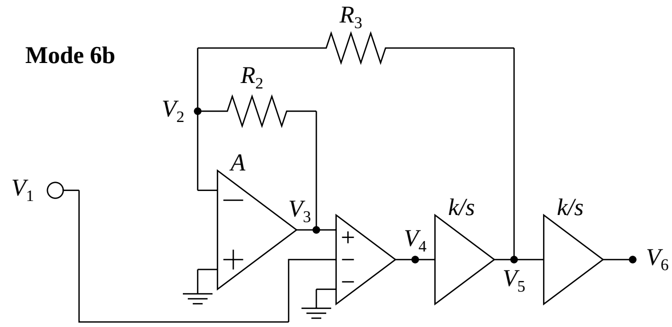

Figure 4.54 Mode 6b SC first-order realization.

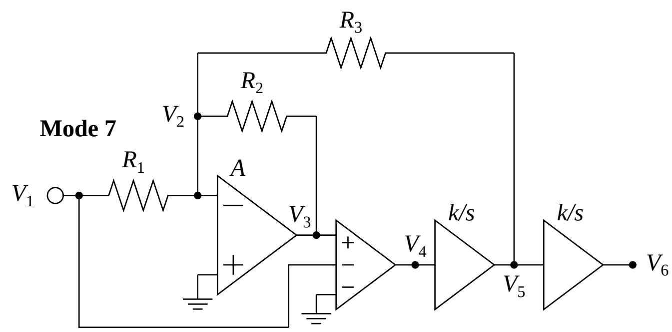

Figure 4.55 Mode 7 SC first-order realization.

The output signal of an SC filter section can be taken at the output of the operational amplifier, V_3, at the output of the first integrator, V_5, and at the output of the second integrator, V_6. The output signal of the three-input summing stage is not available. Modes of operation 6 and 7 do not make use of the output signal V_6. The signal V_1 is the input.

In all modes of operation, the ratio V_6/V_1 is the lowpass transfer function, and V_5/V_1 is the bandpass or lowpass transfer function,

Mode	$\dfrac{V_6}{V_1}$	$\dfrac{V_5}{V_1}$
1, 1a, 1b, 1c, 1d, 2, 2a, 2b, 3, 3a, 4, 4a, 5	$H_{LP} = K\dfrac{\omega_p^2}{s^2 + \dfrac{\omega_p}{Q_p}s + \omega_p^2}$	$H_{BP} = K\dfrac{\dfrac{\omega_p}{Q_p}s}{s^2 + \dfrac{\omega_p}{Q_p}s + \omega_p^2}$
6a, 6b, 7	Not applicable	$H_{LP} = K\dfrac{\omega_p}{s + \omega_p}$

(4.134)

The pole magnitude of an SC section can be $\omega_p = \frac{2\pi}{P} f_{CLK}$ (modes 1, 1a, 1d, 4), $\omega_p > \frac{2\pi}{P} f_{CLK}$ (modes 1c, 2, 2a, 4, 5), or $\omega_p < \frac{2\pi}{P} f_{CLK}$ (mode 1b), or it can be set to an arbitrary value (modes 2b, 3, 3a, 4a).

Most modes of operation are used without any additional operational amplifiers (modes 1, 1a, 1b, 1c, 2, 2a, 2b, 3, 4, 5), while some modes (modes 1d, 3a, 4a) require an additional operational amplifier. In practice, this additional amplifier is often the operation amplifier of the next biquad in the cascade. Thus, the whole filter can be realized with no external operational amplifier.

The output of the operational amplifier can be used for realizing the highpass, bandreject, and allpass transfer functions:

Mode	$H_{HP} = \dfrac{V_3}{V_1} = K \dfrac{s^2}{s^2 + \dfrac{\omega_p}{Q_p} s + \omega_p^2}$
3	$\omega_p = \dfrac{2\pi f_{CLK}}{P} \sqrt{\dfrac{R_2}{R_4}}$

(4.135)

Mode	$H_{BR} = \dfrac{V_3}{V_1} = K \dfrac{s^2 + \omega_p^2}{s^2 + \dfrac{\omega_p}{Q_p} s + \omega_p^2}$
1	$\omega_p = \dfrac{2\pi f_{CLK}}{P}$
1b	$\omega_p = \dfrac{2\pi f_{CLK}}{P} \sqrt{\dfrac{R_6}{R_5 + R_6}}$
1c	$\omega_p = \dfrac{2\pi f_{CLK}}{P} \sqrt{1 + \dfrac{R_6}{R_5 + R_6}}$

(4.136)

Mode	$H_{AP} = \dfrac{V_3}{V_1} = K \dfrac{s^2 - \dfrac{\omega_p}{Q_p} s + \omega_p^2}{s^2 + \dfrac{\omega_p}{Q_p} s + \omega_p^2}$
4	$\omega_p = \dfrac{2\pi f_{CLK}}{P}$
4a	$\omega_p = \dfrac{2\pi f_{CLK}}{P} \sqrt{\dfrac{R_2}{R_4}}$

(4.137)

Mode	$H_{BP} = \frac{V_3}{V_1} = K \frac{\frac{\omega_p}{Q_p}s}{s^2 + \frac{\omega_p}{Q_p}s + \omega_p^2}$
1a	$\omega_p = \frac{2\pi f_{CLK}}{P}$

(4.138)

The highpass notch transfer function can be obtained without an additional amplifier:

Mode	$H_{HPN} = \frac{V_3}{V_1} = K \frac{s^2 + \omega_z^2}{s^2 + \frac{\omega_p}{Q_p}s + \omega_p^2}$	$\frac{R_2}{R_4} > \frac{R_h}{R_l}$
2	$\omega_p = \frac{2\pi f_{CLK}}{P}\sqrt{1 + \frac{R_2}{R_4}}$	$\omega_z = \frac{2\pi f_{CLK}}{P}$
2a	$\omega_p = \frac{2\pi f_{CLK}}{P}\sqrt{\frac{R_2}{R_4} + \frac{R_5 + 2R_6}{R_5 + R_6}}$	$\omega_z = \frac{2\pi f_{CLK}}{P}\sqrt{\frac{R_5 + 2R_6}{R_5 + R_6}}$
2b	$\omega_p = \frac{2\pi f_{CLK}}{P}\sqrt{\frac{R_2}{R_4} + \frac{R_6}{R_5 + R_6}}$	$\omega_z = \frac{2\pi f_{CLK}}{P}\sqrt{\frac{R_6}{R_5 + R_6}}$
3a	$\omega_p = \frac{2\pi f_{CLK}}{P}\sqrt{\frac{R_2}{R_4}}$	$\omega_z = \frac{2\pi f_{CLK}}{P}\sqrt{\frac{R_h}{R_l}}$

(4.139)

while the lowpass notch transfer function requires an additional op amp:

Mode	$H_{LPN} = \frac{V_3}{V_1} = K \frac{s^2 + \omega_z^2}{s^2 + \frac{\omega_p}{Q_p}s + \omega_p^2}$	$\frac{R_2}{R_4} < \frac{R_h}{R_l}$
1d	$\omega_p = \frac{2\pi f_{CLK}}{P}$	$\omega_z = \frac{2\pi f_{CLK}}{P}\sqrt{1 + \frac{R_h}{R_l}}$
3a	$\omega_p = \frac{2\pi f_{CLK}}{P}\sqrt{\frac{R_2}{R_4}}$	$\omega_z = \frac{2\pi f_{CLK}}{P}\sqrt{\frac{R_h}{R_l}}$

(4.140)

The highpass, lowpass, and allpass first-order transfer functions can be obtained using the output of the operational amplifier:

Mode	Transfer function	Pole magnitude
6a	$H_{HP} = \dfrac{V_3}{V_1} = K\dfrac{s}{s + \omega_p}$	$\omega_p = \dfrac{2\pi f_{CLK}}{P}\dfrac{R_2}{R_3}$
6b	$H_{LP} = \dfrac{V_3}{V_1} = K\dfrac{\omega_p}{s + \omega_p}$	$\omega_p = \dfrac{2\pi f_{CLK}}{P}\dfrac{R_2}{R_3}$
7	$H_{AP} = \dfrac{V_3}{V_1} = K\dfrac{s - \omega_p}{s + \omega_p}$	$\omega_p = \dfrac{2\pi f_{CLK}}{P}\dfrac{R_2}{R_3}$

(4.141)

Realizations in mode 6 and 7 do not use the second integrator.

4.9.1 Mode 1 SC Realization

The main feature of the mode 1 operation and its derivatives (modes 1a, 1b, 1c, 1d) is that they do not use the feedback from the second integrator to the operational amplifier. Thus, the gain-sensitivity product of the pole magnitude to the gain of the operational amplifier is zero. This property enables implementation of SC filters at higher frequencies.

Mode 1 requires only three resistors, while mode 1a can be realized with only two external resistors.

4.9.2 Mode 2 SC Realization

The main characteristic of the mode 2 operation and its derivatives (2a, 2b) is that they can realize the highpass notch transfer function without an additional amplifier. The magnitude of the transfer-function zero, ω_z, can be tuned independently of the pole magnitude, ω_p.

Mode 2 requires only four external resistors.

4.9.3 Mode 3 SC Realization

The mode 3 operation is intended for highpass biquads. The pole magnitude, ω_p, can be adjusted to an arbitrary value.

Mode 3a requires an additional operational amplifier. This mode is frequently used for realization of lowpass and highpass notch transfer functions. The magnitude of the transfer-function zero, ω_z, can be adjusted to be higher or lower than the pole magnitude, ω_p.

The notch transfer functions are realized by summing the highpass, V_3, and the lowpass, V_6, outputs using an external op amp and three external resistors.

Mode 3a can realize lowpass notch and highpass notch transfer functions

$$H_{LPN} = K\frac{s^2 + \omega_z^2}{s^2 + \dfrac{\omega_p}{Q_p}s + \omega_p^2}, \quad \omega_z > \omega_p$$
$$H_{HPN} = K\frac{s^2 + \omega_z^2}{s^2 + \dfrac{\omega_p}{Q_p}s + \omega_p^2}, \quad \omega_z < \omega_p \tag{4.142}$$

The resistor values are functions of the transfer-function parameters K, Q_p, ω_p, and ω_z, the clock frequency f_{CLK}, the parameter P, and the resistances R_{1x}, R_{2x}, and R_{hx}. We set the value of R_{1x}, R_{2x}, R_{hx}, and P and compute the element values as

$$\boxed{\begin{aligned}
R_1 &= R_1(K, Q_p, \omega_p, \omega_z, f_{CLK}, P, R_{1x}, R_{2x}, R_{hx}) &&= R_{1x}\\
R_2 &= R_2(K, Q_p, \omega_p, \omega_z, f_{CLK}, P, R_{1x}, R_{2x}, R_{hx}) &&= R_{2x}\\
R_4 &= R_4(K, Q_p, \omega_p, \omega_z, f_{CLK}, P, R_{1x}, R_{2x}, R_{hx}) &&= R_{2x}\left(\frac{2\pi f_{CLK}}{P\omega_p}\right)^2\\
R_3 &= R_3(K, Q_p, \omega_p, \omega_z, f_{CLK}, P, R_{1x}, R_{2x}, R_{hx}) &&= Q_p\sqrt{R_{2x}R_4}\\
R_h &= R_h(K, Q_p, \omega_p, \omega_z, f_{CLK}, P, R_{1x}, R_{2x}, R_{hx}) &&= R_{hx}\\
R_l &= R_l(K, Q_p, \omega_p, \omega_z, f_{CLK}, P, R_{1x}, R_{2x}, R_{hx}) &&= R_{hx}\left(\frac{2\pi f_{CLK}}{P\omega_z}\right)^2\\
R_g &= R_g(K, Q_p, \omega_p, \omega_z, f_{CLK}, P, R_{1x}, R_{2x}, R_{hx}) &&= \frac{K R_{hx} R_{1x}}{R_{2x}}
\end{aligned}} \tag{4.143}$$

The operational amplifier of the next biquad in the cascade can be used instead of the external op amp. In order to avoid the external op amp for the last biquad in the cascade, the highest zero magnitude of the lowpass notch biquad can be moved to infinity, transforming the last biquad into a lowpass biquad.

4.9.4 Mode 4 SC Realization

Modes 4 and 4a are intended for allpass biquads. The pole magnitude of the mode 4 biquad is $\omega_p = \frac{2\pi}{P} f_{CLK}$, while the pole magnitude for mode 4a can be set to an arbitrary value.

The mode 4a operation requires an additional amplifier.

4.9.5 Mode 5 SC Realization

The mode 5 operation can realize second-order transfer functions of the form

$$H_{CZ} = K\frac{s^2 + \dfrac{\omega_z}{Q_z}s + \omega_z^2}{s^2 + \dfrac{\omega_p}{Q_p}s + \omega_p^2}, \qquad \omega_z < \omega_p \tag{4.144}$$

where

$$\begin{aligned}
\omega_p &= \frac{2\pi f_{CLK}}{P}\sqrt{1+\frac{R_2}{R_4}} \\
\omega_z &= \frac{2\pi f_{CLK}}{P}\sqrt{1-\frac{R_1}{R_4}} \\
Q_p &= \frac{R_3}{R_4}\sqrt{1+\frac{R_2}{R_4}} \\
Q_z &= \frac{R_3}{R_1}\sqrt{1-\frac{R_1}{R_4}}
\end{aligned} \tag{4.145}$$

Obviously, the zero Q-factor, Q_z, can be different from the pole Q-factor, Q_p, and the zero magnitude, ω_z, is lower than the pole magnitude, ω_p.

4.9.6 Modes 6 and 7 SC Realizations

By using only the first integrator, modes 6 and 7 realize first-order transfer functions that appear in odd-order filters. The output of the operational amplifier is used for realizing highpass (mode 6a), lowpass (mode 6b), and allpass (mode 7) transfer functions.

4.9.7 Low-Sensitive Lowpass Notch Realization

The extreme of the magnitude response sensitivity of a biquad to the Q -factor, $S_{Q_p}^{M(\omega)}$, is

$$S_{Q_p}^{M(\omega)}(\omega)\Big|_{\max} = 1 \tag{4.146}$$

In the frequency range $0 \le \omega \le \omega_p$ the maximal value of $\left|S_{\omega_p}^{M(\omega)}(\omega)\right|$ is Q_p times larger than the maximal value of $\left|S_{Q_p}^{M(\omega)}(\omega)\right|$:

$$\left|S_{\omega_p}^{M(\omega)}(\omega)\right|_{\max} \approx Q_p\left|S_{Q_p}^{M(\omega)}(\omega)\right|_{\max} \tag{4.147}$$

The upper limit of the magnitude response relative variation of SC filters can be approximated by

$$\left.\frac{\Delta M(\omega)}{M(\omega)}\right|_{\text{worst case}} = \sum_i \left|S_{\omega_p}^{M(\omega)} S_{x_i}^{\omega_p}\frac{\Delta x_i}{x_i}\right| + \sum_i \left|S_{Q_p}^{M(\omega)} S_{x_i}^{Q_p}\frac{\Delta x_i}{x_i}\right| \tag{4.148}$$

where x_i is a resistance or the clock frequency. In practice, the relative variation of the clock frequency is several times smaller than the relative variation of the resistance of the resistors, and it can be neglected in Eq. (4.148). We can assume that all relative variations are the same $\frac{\Delta x_i}{x_i} = \cdots = \frac{\Delta x_j}{x_j}$. Also, we can substitute the approximate maximal values from Eqs. (4.146) and (4.147) into Eq. (4.148):

$$\left.\frac{\Delta M(\omega)}{M(\omega)}\right|_{\text{worst case}} = \left(\sum_i Q_p \left|S_{x_i}^{\omega_p}\right| + \sum_i \left|S_{x_i}^{Q_p}\right|\right) \frac{\Delta x_i}{x_i} \tag{4.149}$$

For most SC filters, $S_{x_i}^{Q_p}$ is 1 or lower. Therefore, for high Q-factor filters, the dominant term can be $Q_p \left|S_{x_i}^{\omega_p}\right|$.

In almost all modes of operation, except modes 1 and 4, $S_{x_i}^{\omega_p} = \frac{1}{2}$. In modes 1 and 4, $S_{x_i}^{\omega_p} = 0$. Mode 1 is obviously superior to other modes with respect to the sensitivity. Nevertheless, this mode is used more in theory than in practice because different clock frequencies must be provided for biquads with different pole magnitudes. So, we could benefit from mode 1 if we could design an SC filter with constant pole magnitudes. In that case we could use only one clock frequency and achieve a very robust design exhibiting the very low passive sensitivity.

We propose an efficient solution to the mode 1 SC filter design based on special elliptic transfer functions which have poles on a circle centered at the origin of the complex s-plane [47].

The lowpass elliptic transfer function can be decomposed into a product of second-order notch transfer functions:

$$H_{LPN}(s) = K \frac{s^2 + \omega_z^2}{s^2 + \dfrac{\omega_p}{Q_p} s + \omega_p^2}, \qquad \omega_z > \omega_p \tag{4.150}$$

The pole magnitudes of all sections are equal to the geometric mean of the passband edge and stopband edge frequency:

$$\omega_p = 2\pi \sqrt{f_{pass} f_{stop}}$$

Therefore, the maximal magnitude response deviation occurs at the frequency $\sqrt{f_{pass} f_{stop}}$, which is in the transition band. Practically, the passband variation is insensitive to the changes of external resistor values.

We propose mode 1d [46] for realization of the notch transfer function (4.150). The transfer function of the mode 1d biquad is

$$H_{LPN}(s) = \frac{R_g R_2}{R_1 R_h} \frac{s^2 + \omega_p^2 \left(1 + \dfrac{R_h}{R_l}\right)}{s^2 + \dfrac{R_2}{R_3} \omega_p s + \omega_p^2} \tag{4.151}$$

The pole Q-factor depends on two resistors

$$Q_p = \frac{R_3}{R_2} \tag{4.152}$$

and the notch frequency is given by

$$\omega_z = \frac{2\pi f_{CLK}}{P}\sqrt{1 + \frac{R_h}{R_l}} \tag{4.153}$$

The magnitude response of notch filters is sensitive not only to ω_p and Q_p but also to ω_z:

$$\left.\frac{\Delta M(\omega)}{M(\omega)}\right|_{\text{worst case}} = \sum_i \left| S_{\omega_z}^{M(\omega)} S_{x_i}^{\omega_z} \frac{\Delta x_i}{x_i} \right| + \sum_i \left| S_{\omega_p}^{M(\omega)} S_{x_i}^{\omega_p} \frac{\Delta x_i}{x_i} \right| + \sum_i \left| S_{Q_p}^{M(\omega)} S_{x_i}^{Q_p} \frac{\Delta x_i}{x_i} \right| \tag{4.154}$$

It has been shown that $S_{\omega_z}^{M(\omega)}$ tends to infinity if ω_z approaches ω_p. In mode 1d the sensitivity $S_{x_i}^{\omega_z}$ is zero for $\omega_z = \omega_p$. The maximal value of the product $\left| S_{\omega_z}^{M(\omega)} S_{x_i}^{\omega_z} \right|$ is $\frac{1}{2}$. Therefore, the zeros of the magnitude response are practically insensitive to small changes of the resistor values.

The resistor values are functions of the transfer-function parameters K, Q_p, ω_p, and ω_z, the clock frequency f_{CLK}, the parameter P, and the resistances R_{1x}, R_{2x}, and R_{hx}. We set the value of R_{1x}, R_{2x}, R_{hx}, and P and compute the element values as

$$\boxed{\begin{aligned}
R_1 &= R_1(K, Q_p, \omega_z, f_{CLK}, P, R_{1x}, R_{2x}, R_{hx}) &&= R_{1x} \\
R_2 &= R_2(K, Q_p, \omega_z, f_{CLK}, P, R_{1x}, R_{2x}, R_{hx}) &&= R_{2x} \\
R_3 &= R_3(K, Q_p, \omega_z, f_{CLK}, P, R_{1x}, R_{2x}, R_{hx}) &&= Q_p R_{2x} \\
R_h &= R_h(K, Q_p, \omega_z, f_{CLK}, P, R_{1x}, R_{2x}, R_{hx}) &&= R_{hx} \\
R_l &= R_l(K, Q_p, \omega_z, f_{CLK}, P, R_{1x}, R_{2x}, R_{hx}) &&= \frac{R_h}{\left(\dfrac{\omega_z P}{2\pi f_{CLK}}\right)^2 - 1} \\
R_g &= R_g(K, Q_p, \omega_z, f_{CLK}, P, R_{1x}, R_{2x}, R_{hx}) &&= \frac{K R_{hx} R_{1x}}{R_{2x}}
\end{aligned}} \tag{4.155}$$

The pole magnitudes of all cascaded biquads are identical, and we are allowed to use single clock-frequency. This way we minimize the influence of resistor values on the pole magnitudes.

4.9.8 Programmable Lowpass/Highpass SC Filters

Generally, our target is to design a programmable cost-effective filter with reduced complexity. A simple modification of mode 3a of a universal SC filter accomplishes this task. A double switch, controlled by an external signal, is introduced to enable the SC biquad to operate as a lowpass or highpass filter.

The second-order lowpass and highpass notch transfer functions can be obtained using mode 3a:

$$
\begin{array}{ll}
\text{Lowpass filter} & \text{Highpass filter} \\
H_{LP}(s) = k_l \dfrac{s^2 + \omega_{lz}^2}{s^2 + \dfrac{\omega_{lp}}{Q_{lp}}s + \omega_{lp}^2} & H_{HP}(s) = k_h \dfrac{s^2 + \omega_{hz}^2}{s^2 + \dfrac{\omega_{hp}}{Q_{hp}}s + \omega_{hp}^2} \\
Q_{lp} = \dfrac{R_3}{\sqrt{R_2 R_4}} & Q_{hp} = \dfrac{R_3'}{\sqrt{R_2' R_4'}} \\
\omega_{lp} = \dfrac{2\pi f_{CLK}}{100}\sqrt{\dfrac{R_2}{R_4}} & \omega_{hp} = \dfrac{2\pi f_{CLK}}{100}\sqrt{\dfrac{R_2'}{R_4'}} \\
\omega_{lz} = \dfrac{2\pi f_{CLK}}{100}\sqrt{\dfrac{R_h}{R_l}} & \omega_{hz} = \dfrac{2\pi f_{CLK}}{100}\sqrt{\dfrac{R_h'}{R_l'}} \\
k_l = \dfrac{R_g R_2}{R_h R_1} & k_h = \dfrac{R_g' R_2'}{R_h' R_1'}
\end{array}
\tag{4.156}
$$

where R_1, R_2, R_3, R_4, R_g, R_h, and R_l are external resistors of the lowpass biquad and R_1', R_2', R_3', R_4', R_g', R_h', and R_l' are external resistors of the highpass biquad.

If we substitute the complex frequency s in the lowpass notch transfer function with $4\pi^2 F_p F_s \dfrac{1}{s}$, which we symbolically designate by $s \rightarrow 4\pi^2 F_p F_s / s$, we obtain the transfer function of a highpass notch filter:

$$
H_{HP}(s) = H_{LP}\left(\frac{4\pi^2 F_p F_s}{s}\right) = k_l \frac{\left(4\pi^2 F_p F_s\right)^2 + \omega_{lz}^2 s^2}{\left(4\pi^2 F_p F_s\right)^2 + 4\pi^2 F_p F_s \dfrac{\omega_{lp}}{Q_{lp}} s + \omega_{lp}^2 s^2} \tag{4.157}
$$

or

$$
H_{HP}(s) = k_l \frac{\omega_{lz}^2}{\omega_{lp}^2} \frac{s^2 + \dfrac{\left(4\pi^2 F_p F_s\right)^2}{\omega_{lz}^2}}{s^2 + \dfrac{4\pi^2 F_p F_s}{\omega_{lp} Q_{lp}} s + \dfrac{\left(4\pi^2 F_p F_s\right)^2}{\omega_{lp}^2}} \tag{4.158}
$$

It is important to notice that $|H_{LP}(j2\pi F_p)| = |H_{HP}(j2\pi F_s)|$, and $|H_{LP}(j2\pi F_s)| = |H_{HP}(j2\pi F_p)|$; this means that the passband edge frequency, F_p, of the lowpass filter is equal to the stopband edge frequency of the highpass filter; the stopband edge frequency, F_s, of the lowpass filter is equal to the passband edge frequency of the highpass filter.

By equating the highpass transfer function from Eqs. (4.156) to (4.158), we find the parameters of the highpass transfer functions k_h, Q_{hp}, ω_{hp}, and ω_{hz}:

$$\omega_{hz} = \frac{4\pi^2 F_p F_s}{\omega_{lz}} \tag{4.159}$$

$$\omega_{hp} = \frac{4\pi^2 F_p F_s}{\omega_{lp}} \tag{4.160}$$

$$Q_{hp} = Q_{lp} \tag{4.161}$$

$$k_h = k_l \frac{\omega_{n,i}^2}{\omega_{0,i}^2} \tag{4.162}$$

and

$$\left(\frac{2\pi f_{CLK}}{100}\right)^2 \sqrt{\frac{R_2'}{R_4'}} = 4\pi^2 F_p F_s \sqrt{\frac{R_4}{R_2}} \tag{4.163}$$

$$\left(\frac{2\pi f_{CLK}}{100}\right)^2 \sqrt{\frac{R_h'}{R_l'}} = 4\pi^2 F_p F_s \sqrt{\frac{R_l}{R_h}} \tag{4.164}$$

If we choose the clock frequency f_{CLK} to meet the condition

$$\left(\frac{f_{CLK}}{100}\right)^2 = F_p F_s \tag{4.165}$$

then the relations between resistors are

$$\frac{R_2'}{R_4'} = \frac{R_4}{R_2} \tag{4.166}$$

$$\frac{R_h'}{R_l'} = \frac{R_l}{R_h} \tag{4.167}$$

Our goal is to use the same set of external resistors to implement both types of filters. We meet this requirement by choosing

$$\begin{aligned} R_2' &= R_4 \\ R_4' &= R_2 \\ R_h' &= R_l \\ R_l' &= R_h \end{aligned} \tag{4.168}$$

According to the above analyses we propose a modification to the mode 3a operation which transforms a lowpass filter to a highpass filter: We have to swap the resistors according to Eq. (4.168).

The modification of the mode 3a operation, which implements a programmable lowpass/highpass filter, is shown in Fig. 4.56. The control signal is applied at input LP/HP, and it controls the double switch which connects the resistors R_2 and R_l or R_4

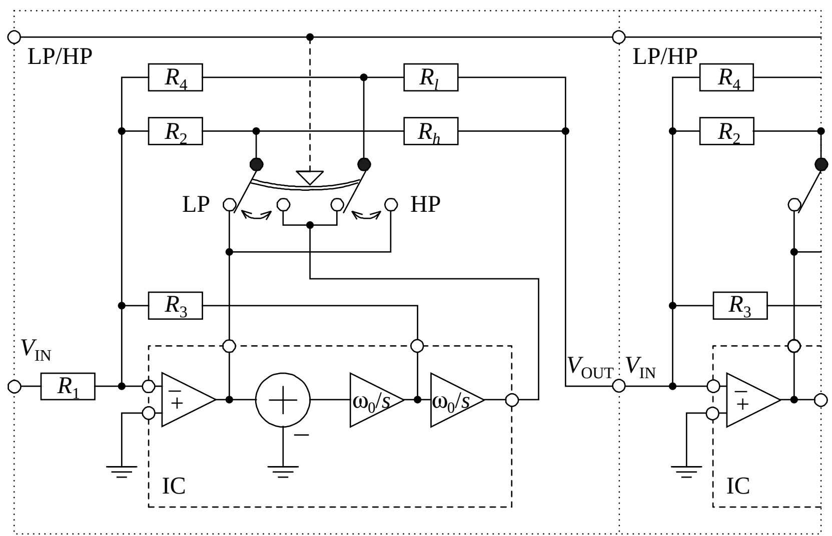

Figure 4.56 Lowpass/highpass programmable SC filter.

and R_h to the output of the summing amplifier, or to the output of the second integrator of the integrated second-order universal SC block.

A simplified second-order section realized with the second-order integrated SC block and resistors R_2, R_3, R_4, R_h and R_l from Fig. 4.56, is shown in Fig. 4.57.

The transfer function of the ith biquad, $T_i(s)$, is defined as the ratio of the output current to the input current:

$$T_i(s) = -\frac{R_2}{R_h}\frac{s^2 + \left(\frac{2\pi f_{CLK}}{100}\right)^2 \frac{R_h}{R_l}}{s^2 + \left(\frac{2\pi f_{CLK}}{100}\right)\frac{R_2}{R_3}s + \left(\frac{2\pi f_{CLK}}{100}\right)^2 \frac{R_2}{R_4}} \tag{4.169}$$

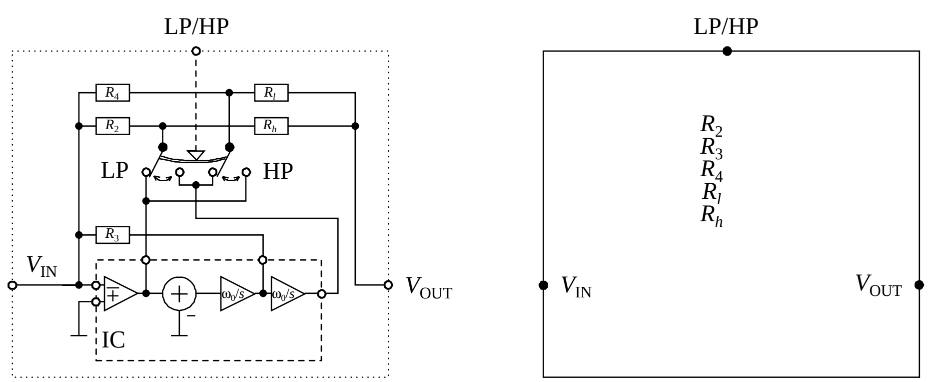

Figure 4.57 Simplified second-order programmable SC filter section.

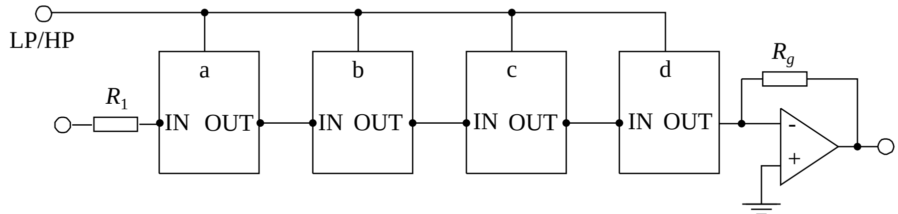

Figure 4.58 Cascade realization of the programmable SC filter.

The cascade realization of the programmable filter is shown in Fig. 4.58. The overall filter transfer function, $H(s)$, is the ratio of the output voltage to the input voltage:

$$H(s) = -\frac{R_g}{R_1}\prod_i T_i(s) \tag{4.170}$$

By changing the external control signal, applied at the LP/HP port, all biquads are simultaneously set to operate as lowpass or highpass filter sections.

4.10 PASSIVE *RLC* FILTERS

In this section we consider realizations of passive RLC filters. We present two basic realizations known as

1. *Singly terminated ladder realization*
2. *Doubly terminated ladder realization*

The doubly terminated RLC filters can be designed to have the lowest sensitivity. These filters consist of capacitors and inductors, and they are terminated at both ends by resistors. The low sensitivity is based on the property that at the frequencies of the magnitude response maxima the generator delivers the maximum power to the filter and to the resistor at the other end which we call the load. Small changes of the capacitors and inductors can slightly change the frequencies of the magnitude response maxima, but these changes do not affect the maximum power delivered to the load. Therefore, the sensitivity must be zero at those frequencies. The frequencies of the magnitude response maxima are in the passband. In the case of elliptic-type filters, as well as in the case of Chebyshev-type filters, those frequencies are distributed over the passband, and thus the sensitivity can not be too large in the passband.

The doubly terminated ladder realization is shown in Fig. 4.59. For $R_g = 0$ this realization becomes the singly terminated ladder realization. The resistor R_g is the output resistance of the voltage generator while R_o is the resistance of the purely resistive load. The transfer function is the ratio of the load voltage and the generator voltage:

$$H(s) = \frac{V_o}{V_g} \tag{4.171}$$

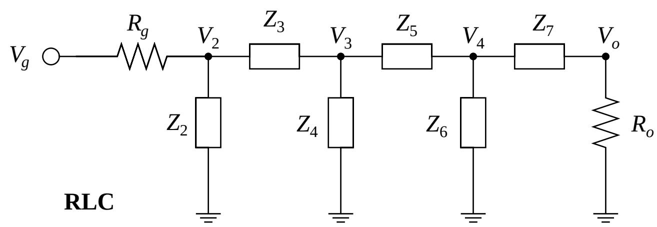

Figure 4.59 Ladder realization.

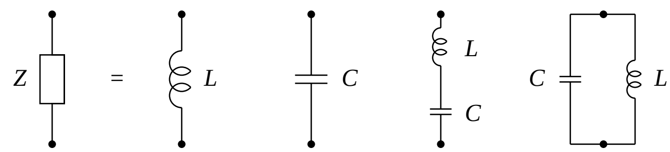

Figure 4.60 Types of ladder branches.

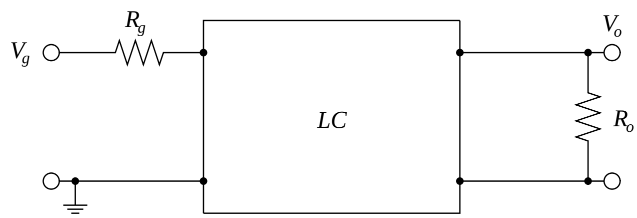

Figure 4.61 Lossless *LC* filter.

A branch of the ladder, whose impedance is Z_i, can be an inductor, a capacitor, the series connection of an inductor and a capacitor, or the shunt connection of an inductor and a capacitor (Fig. 4.60). The first parallel branch can be an open circuit (Z_2 can be omitted from the ladder). The last series branch can be a short circuit ($Z_7 = 0$). The number of ladder branches can be arbitrary.

Any filter that is implemented with capacitors and inductors, only, is called the lossless *LC* filter, because all filter components are (lossless) capacitors and inductors, except the internal resistor of the voltage source (generator), R_g, and the resistive load, R_o. A lossless *LC* filter can be represented by a two-port network, the *LC* network, as shown in Fig. 4.61.

4.10.1 Singly Terminated Ladder Realization

The transfer function of the singly terminated ladder realization, shown in Fig. 4.61, is obtained for $R_g = 0$:

$$H(s) = -\frac{y_{21} R_o}{1 + y_{22} R_o} \tag{4.172}$$

where y_{21} is the ratio of the current through R_o, for $R_o = 0$, to V_g:

$$y_{21} = -\frac{I_{Ro}}{V_g}, \qquad R_o = 0 \tag{4.173}$$

and y_{22} is the ratio of the current through Z_7 to the voltage across R_o, for $V_g = 0$ and $R_o \neq 0$:

$$y_{22} = \frac{I_{Z_7}}{V_{Ro}}, \qquad V_g = 0 \tag{4.174}$$

It can be shown that y_{21} and y_{22} are odd rational functions in complex frequency s (i.e., the ratio of an odd to an even polynomial, or the ratio of an even to an odd polynomial). Therefore, the transfer function of the singly terminated ladder realization is

$$H(s) = \frac{N_{\text{even}} + N_{\text{odd}}}{D_{\text{even}} + D_{\text{odd}}}, \qquad N_{\text{even}} = 0 \quad \text{or} \quad N_{\text{odd}} = 0 \tag{4.175}$$

where N_{even} is the even part and N_{odd} is the odd part of the numerator, while D_{even} and D_{odd} are even and odd part of the denominator, respectively. The transfer function can be rewritten as

$$\begin{aligned} H(s) &= \frac{\dfrac{N_{\text{odd}}}{D_{\text{even}}}}{1 + \dfrac{D_{\text{odd}}}{D_{\text{even}}}}, \quad N_{\text{even}} = 0 \\ H(s) &= \frac{\dfrac{N_{\text{even}}}{D_{\text{odd}}}}{1 + \dfrac{D_{\text{even}}}{D_{\text{odd}}}}, \quad N_{\text{odd}} = 0 \end{aligned} \tag{4.176}$$

yielding

$$y_{21} R_o = \frac{N_{\text{odd}}}{D_{\text{even}}} \quad \text{and} \quad y_{22} R_o = \frac{D_{\text{odd}}}{D_{\text{even}}} \tag{4.177}$$

or

$$y_{21} R_o = \frac{N_{\text{even}}}{D_{\text{odd}}} \quad \text{and} \quad y_{22} R_o = \frac{D_{\text{even}}}{D_{\text{odd}}} \tag{4.178}$$

The admittance y_{22} can be obtained from the transfer function, and it can be realized by the classic Foster or Cauer synthesis procedures. After y_{22} has been realized, we have to determine the generator port, by taking into account the zeros of y_{21}.

The algorithmic details for the singly terminated ladder realization are illustrated by the following examples.

Singly Terminated Ladder Realization with Zeros at the Origin. Let us realize the transfer function

$$H(s) = \frac{s}{s^4 + 3s^3 + 3s^2 + 3s + 1}$$

assuming $R_o = 1\,\Omega$.

First, we find the even and odd parts of the numerator and denominator:

$$N_{\text{odd}} = s$$

$$N_{\text{even}} = 0$$

$$D_{\text{odd}} = 3s^3 + 3s$$

$$D_{\text{even}} = s^4 + 3s^2 + 1$$

Next, we compute y_{21} and y_{22} from

$$y_{21} = \frac{N_{\text{odd}}}{R_o D_{\text{even}}} = \frac{s}{s^4 + 3s^2 + 1}$$

$$y_{22} = \frac{D_{\text{odd}}}{R_o D_{\text{even}}} = \frac{3s^3 + 3s}{s^4 + 3s^2 + 1}$$

Since the order of the y_{22} denominator is larger than the order of the y_{22} numerator, we proceed with the impedance Z_{22}:

$$Z_{22} = \frac{1}{y_{22}} = \frac{s^4 + 3s^2 + 1}{3s^3 + 3s}$$

The impedance Z_{22} is an improper rational function, and we have to extract its polynomial part (by dividing the numerator by the denominator):

$$Z_{22} = \frac{s}{3} + \frac{2s^2 + 1}{3s^3 + 3s}$$

The first term, $\frac{s}{3}$, corresponds to the first serial branch, it is of the form $L_1 s$, and it identifies the inductance of the series inductor L_1:

$$Z_{22} = L_1 s + Z_2, \qquad L_1 = \frac{1}{3}, \qquad Z_2 = \frac{2s^2 + 1}{3s^3 + 3s}$$

The order of the Z_2 denominator is larger than the order of the Z_2 numerator, and we proceed with the admittance

$$Y_2 = \frac{1}{Z_2} = \frac{3s^3 + 3s}{2s^2 + 1}$$

which can be written as (after extracting the polynomial part)

$$Y_2 = \frac{3}{2}s + \frac{\frac{3}{2}s}{2s^2 + 1}$$

The first term, $\frac{3}{2}s$, corresponds to the first parallel branch, it is of the form $C_1 s$, and it identifies the capacitance of the parallel capacitor C_1:

$$Y_2 = C_1 s + Y_3, \qquad C_1 = \frac{3}{2}, \qquad Y_3 = \frac{\frac{3}{2}s}{2s^2 + 1}$$

The order of the Y_3 denominator is larger than the order of the Y_3 numerator, and we proceed with the impedance

$$Z_3 = \frac{1}{Y_3} = \frac{2s^2 + 1}{\frac{3}{2}s}$$

which simplifies to

$$Z_3 = \frac{4}{3}s + \frac{1}{\frac{3}{2}s}$$

and identifies the series connection of an inductor and a capacitor:

$$Z_3 = L_2 s + \frac{1}{C_2 s}, \qquad L_2 = \frac{4}{3}, \qquad C_2 = \frac{3}{2}$$

The LC-ladder realization of y_{22} is shown in Fig. 4.62, and the element values refer to

$$L_1 = \frac{1}{3}\,\text{H}, \qquad C_1 = \frac{3}{2}\,\text{F}, \qquad L_2 = \frac{4}{3}\,\text{H}, \qquad C_2 = \frac{3}{2}\,\text{F}$$

Finally, we have to determine the terminals at which the voltage generator should be connected. The generator can be placed in one of the ladder branches.

The transfer function $H(s)$ has one real zero, $s_z = 0$, and it implies that the generator should be inserted in series with a capacitor. We examine the transfer function for large s, $s \to \infty$:

- If we place the generator in series with C_1, the transfer function $H_1(s) = V_o/V_g$ asymptotically tends to $\frac{1}{s}$, which disagrees with $H(s)$.
- If we place the generator in series with C_2, the transfer function $H_2(s) = V_o/V_g$ asymptotically tends to $\frac{1}{s^3}$, which is in agreement with $H(s)$. This realization is shown in Fig. 4.62.

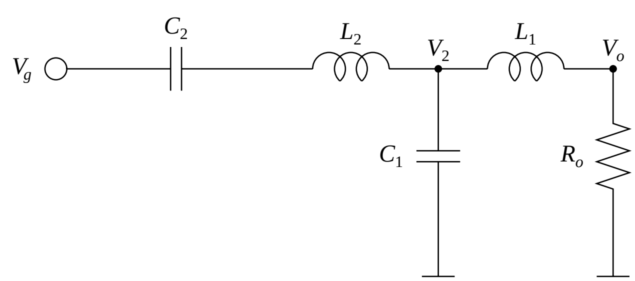

Figure 4.62 Singly terminated LC-ladder realization with zeros at the origin.

Let us derive the transfer function of the circuit from Fig. 4.62 to validate the realization

$$H_2(s) = \frac{\frac{3}{2}s}{s^4 + 3s^3 + 3s^2 + 3s + 1} = \frac{3}{2}H(s)$$

This transfer function has the desired poles and zeros, but the gain constant is different; that is, it is $\frac{3}{2}$ instead of 1. Therefore, the above procedure can realize the desired transfer function within a constant multiplier.

This procedure is applicable only to transfer functions with zeros at the origin.

Singly Terminated Ladder Realization with Complex Zeros. Consider the transfer function with complex zeros:

$$H(s) = \frac{(s^2 + 3.476896154)(s^2 + 8.227391422)}{55.3858\,(s + 0.60913)\left(s^2 + 0.263147s + 1.166357185\right)\left(s^2 + 0.85422659s + 0.7269594794\right)}$$

assuming $R_o = 1\,\Omega$.

First, we find the even and odd parts of the numerator and denominator:

$$N_{\text{odd}} = 0$$

$$N_{\text{even}} = 0.516482303994966 + 0.2113228946047543s^2 + 0.01805516937554391s^4$$

$$D_{\text{odd}} = 1.571315794907603s + 2.79872960375543s^3 + s^5$$

$$D_{\text{even}} = 0.5164779231828084 + 2.477831112275122s^2 + 1.72650359s^4$$

Next, we compute y_{21} and y_{22}:

$$y_{21} = \frac{0.516482303994966 + 0.2113228946047543s^2 + 0.01805516937554391s^4}{1.571315794907603s + 2.79872960375543s^3 + s^5}$$

$$y_{22} = \frac{0.5164779231828084 + 2.477831112275122s^2 + 1.72650359s^4}{1.571315794907603s + 2.79872960375543s^3 + s^5}$$

The transfer function zeros coincide with the zeros of y_{21}:

$$s_1 = +j2.868342974959585$$

$$s_2 = -j2.868342974959585$$

$$s_3 = +j1.864643706985332$$

$$s_4 = -j1.864643706985332$$

or

$$s_1^2 = s_2^2 = -8.2273914225$$

$$s_3^2 = s_4^2 = -3.476896154$$

and these zeros must be realized by the LC ladder derived from y_{22}.

Since the order of the y_{22} denominator is larger than the order of the y_{22} numerator, we proceed with the impedance Z_{22}:

$$Z_{22} = \frac{1}{y_{22}} = \frac{1.5713157949076s + 2.79872960375543s^3 + s^5}{0.5164779231828 + 2.477831112275s^2 + 1.72650359s^4}$$

The impedance Z_{22} is an improper rational function in s, and it can be expressed as a sum of a linear term $L_1 s$ and an impedance Z_2; our goal is to find L_1 such that Z_2 has at least one complex pair of zeros that coincide with the transfer-function zeros (say, the zeros s_1 and s_2):

$$Z_{22} = L_1 s + Z_2, \qquad Z_2(s_1) = Z_2(s_2) = 0$$

The inductance L_1 is computed from

$$L_1 = \left.\frac{Z_{22}}{s}\right|_{s=s_1} = \frac{1.5713157949 + 2.7987296 s_1^2 + s_1^4}{0.516477923 + 2.47783 s_1^2 + 1.7265 s_1^4} = 0.47666285$$

and the impedance Z_2 is found as

$$Z_2 = Z_{22} - L_1 s = s\frac{0.76752227 + 0.93694538 s^2 + 0.102542432 s^4}{0.2991467415 + 1.43517286997 s^2 + s^4}$$

or, equivalently,

$$Z_2 = s\frac{(s^2 + 8.2273914225)(0.093288654 + 0.102542432 s^2)}{0.2991467415 + 1.43517286997 s^2 + s^4}$$

We proceed with the admittance Y_2

$$Y_2 = \frac{1}{Z_2} = \frac{0.2991467415 + 1.43517286997 s^2 + s^4}{s(s^2 + 8.2273914225)(0.093288654 + 0.102542432 s^2)}$$

and extract a biquadratic term with the pole pair s_1, s_2, which corresponds to a simple LC connection:

$$Y_2 = \frac{1}{sL_2 + \dfrac{1}{sC_2}} + Y_3 = \frac{\dfrac{s}{L_2}}{s^2 + \dfrac{1}{C_2 L_2}} + Y_3 = \frac{\dfrac{s}{L_2}}{s^2 + s_1 s_2} + Y_3$$

We find L_2

$$L_2 = Z_2 \left.\frac{s}{s^2 + 8.2273914225}\right|_{s=s_1} = s_1^2 \frac{(0.093288654 + 0.102542432 s_1^2)}{0.2991467415 + 1.43517286997 s_1^2 + s_1^4}$$

$$L_2 = 0.1098864$$

and C_2

$$C_2 = -\frac{1}{L_2 s_1^2} = \frac{1}{8.2273914225 L_2} = 1.1060985$$

The impedance Y_3 can be calculated from

$$Y_3 = Y_2 - \frac{\dfrac{s}{0.1098864}}{s^2 + 8.2273914225}$$

$$= \frac{0.2991467415 + 1.43517286997 s^2 + s^4}{s(s^2 + 8.2273914225)(0.093288654 + 0.102542432 s^2)} - \frac{\dfrac{s}{0.1098864}}{s^2 + 8.2273914225}$$

yielding

$$Y_3 = \frac{0.06683258(0.5440438232 + s^2)}{s(0.093288654 + 0.102542432s^2)} = \frac{0.65175537(0.5440438232 + s^2)}{s(0.909756595 + s^2)}$$

Since the order of the Y_3 denominator is larger than the order of the Y_3 numerator, we proceed with the impedance Z_3:

$$Z_3 = \frac{1}{Y_3} = \frac{s(0.909756595 + s^2)}{0.65175537(0.5440438232 + s^2)}$$

The impedance Z_3 is an improper rational function in s, and it can be expressed as a sum of a linear term $L_3 s$ and an impedance Z_4; our goal is to find L_3 such that Z_4 has at least one complex pair of zeros that coincide with the transfer-function zeros (excluding the zeros s_1 and s_2, but using s_3 and s_4):

$$Z_3 = L_3 s + Z_4$$

We obtain

$$L_3 = \left.\frac{Z_3}{s}\right|_{s=s_3} = \frac{0.909756595 + s_3^2}{0.65175537(0.5440438232 + s_3^2)} = 1.342996$$

and

$$Z_4 = Z_3 - L_3 s = s\frac{0.665207337596 + 0.1913221759s^2}{0.54404382321885 + s^2}$$

$$Z_4 = s\frac{0.1913221759(3.476896154 + s^2)}{0.54404382321885 + s^2}$$

Again, we proceed with

$$Y_4 = \frac{1}{Z_4} = \frac{0.54404382321885 + s^2}{0.1913221759s(3.476896154 + s^2)}$$

and extract a biquadratic term with the pole pair s_3, s_4, which corresponds to a simple LC connection:

$$Y_4 = \frac{1}{sL_4 + \dfrac{1}{sC_4}} + Y_5 = \frac{\dfrac{s}{L_4}}{s^2 + \dfrac{1}{C_4 L_4}} + Y_5 = \frac{\dfrac{s}{L_4}}{s^2 + s_3 s_4} + Y_5$$

We find

$$L_4 = \left. Z_4 \frac{s}{s^2 + 3.476896154}\right|_{s=s_3} = s_3^2 \frac{0.1913221759}{0.54404382321885 + s_3^2} = 0.226812421$$

and

$$C_4 = -\frac{1}{L_4 s_3^2} = \frac{1}{3.476896154 L_4} = 1.2680648$$

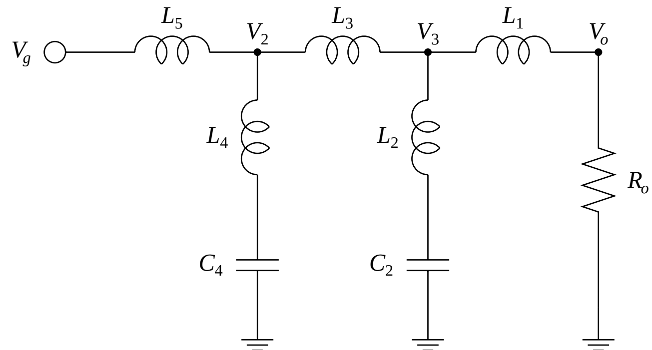

Figure 4.63 Singly terminated LC-ladder realization with complex zeros.

The impedance Y_5 can be calculated from

$$Y_5 = Y_4 - \frac{\frac{s}{L_4}}{s^2 - s_3^2} = \frac{0.54404382321885 + s^2}{0.1913221759s(3.476896154 + s^2)} - \frac{\frac{s}{0.226812421}}{s^2+3.47689615}$$
$$= \frac{0.156474(3.476896154 + s^2)}{0.1913221759s(3.476896154 + s^2)} = \frac{1}{1.22270919s} = \frac{1}{L_5 s}$$

and identified as an admittance of an inductor

$$L_5 = 1.22270919$$

The realization is shown in Fig. 4.63; the element values are

$$L_1 = 0.47666\,\text{H}$$
$$L_2 = 0.10989\,\text{H}$$
$$C_2 = 1.1061\,\text{F}$$
$$L_3 = 1.343\,\text{H}$$
$$L_4 = 0.22681\,\text{H}$$
$$C_4 = 1.268\,\text{F}$$
$$L_5 = 1.2227\,\text{H}$$

The above procedure is known as the *zero shifting technique*.

4.10.2 Doubly Terminated Ladder Networks

The transfer function of the doubly terminated ladder realization, shown in Fig. 4.61, is obtained for $R_g \neq 0$:

$$H(s) = \frac{V_o}{V_g} = \frac{R_o z_{21}}{R_g R_o + R_o z_{11} - z_{12} z_{21} + R_g z_{22} + z_{11} z_{22}} \tag{4.179}$$

where z_{11}, z_{12}, z_{21}, and z_{22} are rational functions in s, known as z-parameters of the LC network, such that

$$\begin{aligned} V_1 &= z_{11}I_1 + z_{12}I_2 \\ V_2 &= z_{21}I_1 + z_{22}I_2 \end{aligned} \tag{4.180}$$

It can be shown that $z_{12} = z_{21}$ holds for passive LC networks; also, z_{11} and z_{22} are odd rational functions in complex frequency s (i.e., the ratio of an odd and even polynomial, or the ratio of an even and odd polynomial).

In order to find the doubly terminated ladder realization whose transfer function is $H(s)$, the first step is to find z_{11} and z_{22} from $H(s)$. Then, we proceed with the realization of z_{11} or z_{22}. The transfer function zeros coincide with the zeros of z_{21} [Eq. (4.179)], so we do not have to find z_{21}.

The basic idea used in deriving z_{11} and z_{22} is that the average power entering the LC network must be equal to the average power leaving the LC network, which is delivered to the load R_o. For this purpose we define auxiliary rational functions $K(s)$ and $\mathcal{K}(\omega^2)$ such that

$$K(s)K(-s)|_{s^2=-\omega^2} = \mathcal{K}(\omega^2) = \mathcal{K}(-s^2) = \frac{M_{\max}^2}{M^2(\omega)} - 1 \tag{4.181}$$

where $M(\omega) = |H(j\omega)|$ is the magnitude response, and $M_{\max} = \max\limits_{\omega} M(\omega)$ is the maximum value of the magnitude response. $\mathcal{K}(\omega^2)$ is an even rational function in ω, $\mathcal{K}(-s^2)$ is an even rational function in s with real coefficients, and

$$\mathcal{K}(\omega^2) \geq 0 \tag{4.182}$$

The complex zeros of $\mathcal{K}(-s^2)$ appear in quadruples, that is, $s_1 = \sigma_1 + j\omega_1$, $s_2 = \sigma_1 - j\omega_1$, $s_3 = -\sigma_1 + j\omega_1$, and $s_4 = -\sigma_1 - j\omega_1$; σ_1, ω_1 are real and positive. Since $K(s)$ is a rational function with real coefficients, we must choose complex-conjugate pairs of zeros from the left, or right, complex s-plane; that is, we can choose s_1 and s_2, or s_3 and s_4. If the zeros are purely imaginary, it is irrelevant which pair we choose.

The real zeros of $\mathcal{K}(-s^2)$ appear in pairs—that is, $s_1 = \sigma_1$ and $s_2 = -\sigma_1$—and we select only one zero for $K(s)$.

The normalized transfer function $H_n(s)$, used in finding the doubly terminated ladder realization,

$$H_n(s) = \frac{H(s)}{M_{\max}} \tag{4.183}$$

is expressed as

$$H_n(s) = \frac{P(s)}{D_{\text{even}} + D_{\text{odd}}} = \frac{P_{\text{even}} + P_{\text{odd}}}{D_{\text{even}} + D_{\text{odd}}} \tag{4.184}$$

where D_{even} is an even polynomial in s, D_{odd} is an odd polynomial in s, and $P(s)$ is determined by the transfer function zeros. P_{even} is an even polynomial in s, and P_{odd} is an odd polynomial in s.

From (4.181) we find

$$K(s) = \frac{N_{\text{even}} + N_{\text{odd}}}{P(s)} = \frac{N_{\text{even}} + N_{\text{odd}}}{P_{\text{even}} + P_{\text{odd}}} \tag{4.185}$$

where N_{even} is an even polynomial in s, and N_{odd} is an odd polynomial in s. We adopt that the poles of $K(s)$ are the transfer function zeros.

It can be shown [24] that

$$z_{11} = R_g \frac{D_{\text{even}} - N_{\text{even}}}{D_{\text{odd}} + N_{\text{odd}}}, \qquad z_{22} = R_o \frac{D_{\text{even}} + N_{\text{even}}}{D_{\text{odd}} + N_{\text{odd}}} \tag{4.186}$$

or

$$z_{11} = R_g \frac{D_{\text{odd}} - N_{\text{odd}}}{D_{\text{even}} + N_{\text{even}}}, \qquad z_{22} = R_o \frac{D_{\text{odd}} + N_{\text{odd}}}{D_{\text{even}} + N_{\text{even}}} \tag{4.187}$$

The impedance z_{11} can be realized by the classical Foster or Cauer synthesis procedures.

The algorithmic details for the doubly terminated ladder realization are illustrated by the following example.

Doubly Terminated Ladder Realization with Complex Zeros. Consider the transfer function with complex zeros:

$$H(s) = \frac{(s^2 + 3.476896154)(s^2 + 8.227391422)}{55.3858\,(s + 0.60913)\left(s^2 + 0.263147s + 1.166357185\right)\left(s^2 + 0.85422659s + 0.7269594794\right)}$$

assuming $R_g = R_o = 1\,\Omega$.

The magnitude response maximum is

$$M_{\max} = \max_{\omega} |H(j\omega)| = 1.00000848$$

The normalized transfer function is given by

$$H_n(s) = \frac{H(s)}{1.00000848}$$

and expressed as

$$H_n(s) = \frac{P_{\text{even}} + P_{\text{odd}}}{D_{\text{even}} + D_{\text{odd}}} \tag{4.188}$$

with

$$\begin{aligned}
P_{\text{odd}} &= 0 \\
P_{\text{even}} &= 28.6058 + 11.7043s^2 + s^4 \\
D_{\text{odd}} &= 87.0293s + 155.011s^3 + 55.3863s^5 \\
D_{\text{even}} &= 28.6058 + 137.238s^2 + 95.6246s^4
\end{aligned}$$

The auxiliary function $\mathcal{K}(-s^2)$ is computed from

$$\mathcal{K}(-s^2) = \frac{D_{\text{even}}^2 - D_{\text{odd}}^2}{P_{\text{even}}^2 - P_{\text{odd}}^2} - 1 = \frac{D_{\text{even}}^2 - D_{\text{odd}}^2 - P_{\text{even}}^2 + P_{\text{odd}}^2}{P_{\text{even}}^2 - P_{\text{odd}}^2} = \frac{N_{\text{even}}^2 - N_{\text{odd}}^2}{P_{\text{even}}^2 - P_{\text{odd}}^2} \tag{4.189}$$

The 10 zeros of $\mathcal{K}(-s^2)$, obtained from the algebraic equation in s,

$$D_{\text{even}}^2 - D_{\text{odd}}^2 - P_{\text{even}}^2 + P_{\text{odd}}^2 = 0$$

are

$$
\begin{aligned}
s_{k1} &= 0 & s_{k6} &= 0 \\
s_{k2} &= -0.00102181 + 0.95906j & s_{k7} &= 0.00102181 + 0.95906j \\
s_{k3} &= -0.00102181 - 0.95906j & s_{k8} &= 0.00102181 - 0.95906j \\
s_{k4} &= -0.00284197 + 0.623457j & s_{k9} &= 0.00284197 + 0.623457j \\
s_{k5} &= -0.00284197 - 0.623457j & s_{k10} &= 0.00284197 - 0.623457j
\end{aligned}
$$

The five zeros of $K(s)$ can be summarized as

$$
\begin{aligned}
s_{k1} &= 0 \\
s_{k2} &= \alpha_1 0.00102181 + 0.95906j \\
s_{k3} &= \alpha_1 0.00102181 - 0.95906j \\
s_{k4} &= \alpha_2 0.00284197 + 0.623457j \\
s_{k5} &= \alpha_2 0.00284197 - 0.623457j
\end{aligned}
\qquad
\begin{aligned}
\alpha_1 &\in \{1, -1\} \\
\alpha_2 &\in \{1, -1\}
\end{aligned}
$$

where the real part can be positive for $\alpha_i = 1$ or negative for $\alpha_i = -1$.

Let us select $\alpha_1 = 1$ and $\alpha_2 = 1$; we find $K(s)$ and the even and odd parts of the numerator of $K(s)$ and the denominator of $H_n(s)$:

$$
\begin{aligned}
N_{\text{odd}} &= 0.357531s + 1.30852s^3 + s^5 \\
N_{\text{even}} &= 0.00602244s^2 + 0.00772756s^4 \\
D_{\text{odd}} &= 1.57132s + 2.79873s^3 + s^5 \\
D_{\text{even}} &= 0.516478 + 2.47783s^2 + 1.7265s^4
\end{aligned}
$$

New D_{odd} and D_{even} are obtained after normalizing the numerator of $H_n(s)$ in such a way that the coefficient of the highest term, s^5, is unity.

Using Eq. (4.186), we find z_{11}:

$$z_{11} = R_g \frac{D_{\text{even}} - N_{\text{even}}}{D_{\text{odd}} + N_{\text{odd}}} = \frac{0.516478 + 2.47181s^2 + 1.71878s^4}{1.92885s + 4.10724s^3 + 2s^5}$$

The transfer function zeros are

$$
\begin{aligned}
s_1 &= +j2.868342974959585 \\
s_2 &= -j2.868342974959585 \\
s_3 &= +j1.864643706985332 \\
s_4 &= -j1.864643706985332
\end{aligned}
$$

or

$$
\begin{aligned}
s_1^2 = s_2^2 &= -8.2273914225 \\
s_3^2 = s_4^2 &= -3.476896154
\end{aligned}
$$

and they must be realized by the LC ladder derived from z_{11}.

Since the order of the z_{11} denominator is larger than the order of the z_{11} numerator, we proceed with the admittance Y_{11}:

$$Y_{11} = \frac{1}{z_{11}} = \frac{1.92885s + 4.10724s^3 + 2s^5}{0.516478 + 2.47181s^2 + 1.71878s^4}$$

The admittance Y_{11} is an improper rational function in s, and it can be expressed as a sum of a linear term $C_1 s$ and an admittance Y_2; our goal is to find C_1 such that Y_2 has at least one complex pair of zeros that coincide with the transfer-function zeros (say, the zeros s_1 and s_2):

$$Y_{11} = C_1 s + Y_2, \qquad Y_2(s_1) = Y_2(s_2) = 0$$

The capacitance C_1 is computed from

$$C_1 = \left.\frac{Y_{11}}{s}\right|_{s=s_1} = \frac{1.92885 + 4.10724s^2 + 2s^4}{0.516478 + 2.47181s^2 + 1.71878\,s^4} = 1.07245$$

and the admittance Y_2 is found as

$$Y_2 = Y_{11} - C_1 s = s\frac{1.37495 + 1.45636s^2 + 0.156701s^4}{0.516478 + 2.47181s^2 + 1.71878s^4}$$

or, equivalently,

$$Y_2 = s\frac{0.0911699\left(8.22739 + s^2\right)\left(1.06648 + s^2\right)}{\left(0.253704 + s^2\right)\left(1.18442 + s^2\right)}$$

We proceed with the impedance Z_2,

$$Z_2 = \frac{1}{Y_2} = \frac{\left(0.253704 + s^2\right)\left(1.18442 + s^2\right)}{0.0911699s\left(8.22739 + s^2\right)\left(1.06648 + s^2\right)}$$

and extract a biquadratic term with the pole pair s_1, s_2, which corresponds to a simple LC connection

$$Z_2 = \frac{1}{sC_2 + \dfrac{1}{sL_2}} + Z_3 = \frac{\dfrac{s}{C_2}}{s^2 + \dfrac{1}{L_2C_2}} + Z_3 = \frac{\dfrac{s}{C_2}}{s^2 + s_1 s_2} + Z_3$$

We find C_2,

$$C_2 = \left.Y_2\frac{s}{s^2 + 8.2273914225}\right|_{s=s_1} = s_1^2\frac{0.0911699\left(1.06648 + s_1^2\right)}{\left(0.253704 + s_1^2\right)\left(1.18442 + s_1^2\right)}$$
$$C_2 = 0.0956459$$

and L_2,

$$L_2 = -\frac{1}{C_2 s_1^2} = \frac{1}{8.2273914225C_2} = 1.27078$$

The impedance Z_3 can be calculated from

$$Z_3 = Z_2 - \frac{\dfrac{s}{0.0956459}}{s^2+8.2273914225}$$

yielding

$$Z_3 = \frac{0.513303\left(0.78045 + s^2\right)}{s\left(1.06648 + s^2\right)}$$

Since the order of the Z_3 denominator is larger than the order of the Z_3 numerator, we proceed with the admittance Y_3:

$$Y_3 = \frac{1}{Z_3} = \frac{s\left(1.06648 + s^2\right)}{0.513303\left(0.78045 + s^2\right)}$$

The admittance Y_3 is an improper rational function in s, and it can be expressed as a sum of a linear term $C_3 s$ and an admittance Y_4; our goal is to find C_3 such that Y_4 has at least one complex pair of zeros that coincide with the transfer-function zeros (excluding the zeros s_1 and s_2, but using s_3 and s_4):

$$Y_3 = C_3 s + Y_4$$

We obtain

$$C_3 = \left.\frac{Y_3}{s}\right|_{s=s_3} = \frac{1.06648 + s_3^2}{0.513303\left(0.78045 + s_3^2\right)} = 1.74151$$

and

$$Y_4 = Y_3 - C_3 s = s\frac{0.718527 + 0.206658\,s^2}{0.78045 + s^2} = s\frac{0.206658\left(3.4769 + s^2\right)}{0.78045 + s^2}$$

Again, we proceed with

$$Z_4 = \frac{1}{Y_4} = \frac{0.78045 + s^2}{0.206658s\left(3.4769 + s^2\right)}$$

and extract a biquadratic term with the pole pair s_3, s_4, which corresponds to a simple LC connection:

$$Z_4 = \frac{1}{sC_4 + \dfrac{1}{sL_4}} + Z_5 = \frac{\dfrac{s}{C_4}}{s^2 + \dfrac{1}{L_4 C_4}} + Z_5 = \frac{\dfrac{s}{C_4}}{s^2+s_3 s_4} + Z_5$$

We find

$$C_4 = \left.Y_4\frac{s}{s^2 + 3.476896154}\right|_{s=s_3} = s_3^2\frac{0.206658}{0.78045 + s_3^2} = 0.266472$$

and

$$L_4 = -\frac{1}{C_4 s_3^2} = \frac{1}{3.476896154C_4} = 1.07934$$

The impedance Z_5 can be calculated from

$$Z_5 = Z_4 - \frac{\frac{s}{C_4}}{s^2 - s_3^2} = \frac{1.08618}{s} = \frac{1}{C_5 s}$$

and identified as an impedance of a capacitor:

$$C_5 = 0.920658$$

The same procedure can be repeated for all possible combinations of α_1 and α_2. The realization is shown in Fig. 4.64, and the element values for various α_1 and α_2 follow:

α_1	α_2	C_1 (F)	C_2 (F)	C_3 (F)	C_4 (F)	C_5 (F)	L_1 (H)	L_2 (H)
1	1	1.0724	0.095646	1.7415	0.26647	0.92066	1.2708	1.0793
−1	1	1.0695	0.095821	1.7413	0.26590	0.92382	1.2685	1.0817
1	−1	1.0649	0.095526	1.7417	0.26687	0.92796	1.2724	1.0777
−1	−1	1.0620	0.095700	1.7414	0.26629	0.93117	1.2701	1.0801
0	0	1.0672	0.095675	1.7415	0.26639	0.92594	1.2704	1.0797
1%	→	1.07	0.096	1.74	0.27	0.93	1.27	1.08

Approximate element values, which are within 1% error, are given in the last row. The approximate element values are important for implementation because commercial components are available within some tolerances.

Formally, we can examine the case $\alpha_1 = \alpha_2 = 0$, which implies purely imaginary zeros. This is legitimate if the real parts of the zeros are negligibly small, or when we know that the function $K(s)$ has purely imaginary zeros (which is exactly the case when we design elliptic-type filters).

Let us derive the transfer function of the circuit from Fig. 4.64 to validate the realization:

$$H_{\mathrm{val}}(s) = \frac{(3.4769+s^2)(8.22739+s^2)}{110.773\ (0.60913+s)(1.16636+0.263147s+s^2)\ (0.726959+0.854227s+s^2)}$$

$$H_{\mathrm{val}}(s) = \frac{1}{2}H(s)$$

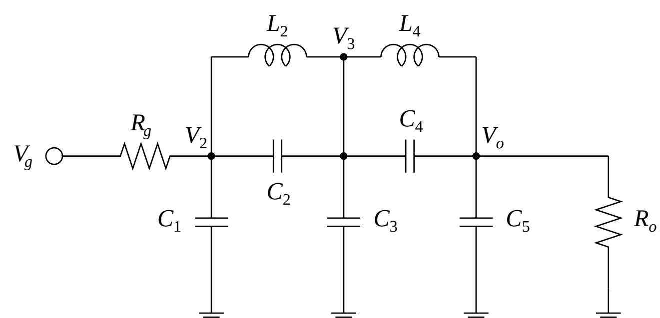

Figure 4.64 Doubly terminated LC-ladder realization with complex zeros.

The transfer function $H_{\text{val}}(s)$ has the desired poles and zeros, but the gain constant is different; that is, it is $\frac{1}{2}$ instead of 1. Therefore, the above procedure can realize the desired transfer function within a constant multiplier.

Instead of using z_{11},

$$z_{11} = R_g \frac{D_{\text{even}} - N_{\text{even}}}{D_{\text{odd}} + N_{\text{odd}}}$$

we could use z_{22},

$$z_{22} = R_o \frac{D_{\text{even}} + N_{\text{even}}}{D_{\text{odd}} + N_{\text{odd}}}$$

to realize the required transfer function $H(s)$. Obviously, the two circuits should be the same. If this is not the case we have to change R_o to provide same element values (capacitances and inductances) in both realizations [24].

4.10.3 Quality Factor of Inductors and Capacitors

For some practical applications a critical quantity might be the quality factor of inductors and capacitors. The quality factor of an inductor, Q_L, and a capacitor, Q_C, is defined as [30]

$$Q_L = \frac{2\pi f L}{R}$$
$$Q_C = \frac{2\pi f C}{G}$$

where L is the inductance in H, C is the capacitance in F, R is the inductor loss resistance in Ω, and G is the capacitor loss conductance in S.

An LC filter with $R = 0$ and $G = 0$ is called the lossless filter. $R > 0$ and $G > 0$ cause dissipation or loss. A transfer-function pole, s_p, of an LC filter implemented with lossy elements ($R > 0$, $G > 0$) can be approximately computed from the transfer-function pole, s_{p0}, of the lossless LC filter from

$$s_p \approx s_{p0} + \frac{1}{2Q_L} + \frac{1}{2Q_C}$$

assuming that the Q-factors of all inductors are Q_L and that the Q-factors of all capacitors are Q_C [30].

In general, $Q_C \gg Q_L$ and we can neglect the capacitor losses. Q_L must be sufficiently larger than the maximal transfer-function pole Q-factor, $Q_{\max}$:

$$Q_L \gg Q_{\max}$$

For practical applications, usually, we require [30]

$$Q_L \geq 3Q_{\max}$$

Lossy components increase the passband attenuation by an amount proportional to the group delay of the lossless filter. Therefore, we prefer the transfer function with lower pole Q-factors and smaller group delays. This fact should be seriously taken into account in the approximation step of the filter design.

4.11 OPERATIONAL TRANSCONDUCTANCE AMPLIFIER (OTA) FILTERS

In this section we consider realizations of continuous-time active RC filters with operational transconductance amplifiers (OTA). There are two types of these filters:

- *OTA-C filters* (integrated filters implemented with OTAs and poly-silicon capacitors)
- *OTA-R-C filters* (integrated filters implemented with OTAs, poly-silicon capacitors, and linear full CMOS resistors)

The actual trend in integrated circuits technology is to incorporate in a chip as many digital and analog blocks as possible. The mixed-mode (analog and digital) integrated circuits may require high-quality analog filters before analog-to-digital signal conversion, as well as after the digital-to-analog signal conversion. In order to use very small silicon area and to manufacture low-cost integrated circuits, it is preferable to use the same technologies for digital and analog circuits. Available technologies for integrated circuits are complementary-metal-oxide-semiconductor (CMOS), bipolar, bipolar-CMOS (BICMOS), gallium-arsenide.

In many communication applications, the frequency range of filters is from a few kHz up to several GHz. The conventional operational amplifier cannot be used as an amplifier at very high frequencies. High-frequency and high-selectivity continuous-time filters demand new active devices acting as amplifiers. The useful frequency range of the operational amplifier is lower than the useful frequency range of OTA.

OTA is basically a voltage-to-current transducer. OTA converts the input voltage $(V_+ - V_-)$ into the output current I_o:

$$I_o = g_m(V_+ - V_-)$$

where g_m is the OTA *transconductance* with units of ampere/volt or siemens, abbreviated S [16 pp. 2472–2490]. Typical values for g_m are tens to hundreds of μS in CMOS technology, and up to mS in bipolar technology.

The most commonly used circuit symbol and a simplified small-signal equivalent circuit for OTA is shown in Fig. 4.65.

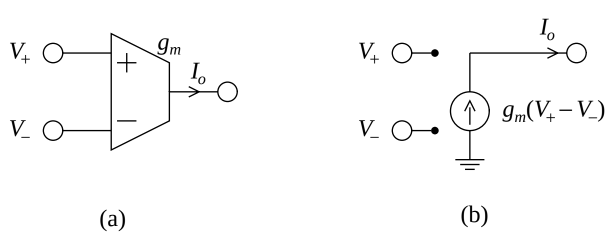

Figure 4.65 (a) OTA circuit symbol, (b) OTA small-signal equivalent cicuit.

Most of the fully integrated OTA filters use OTAs and capacitors as the main components. They are sensitive to parasitic capacitances. If the parasitic capacitances and temperature variations are considered, typically, the tolerances of the ratio $r_{cg} = C/g_m$ can be greater than 30%. However, for a unity capacitor ratio, $r_c = C_1/C_2 \approx 1$, a 0.1% tolerance of r_c can be achieved. The tolerance of the unity transconductance ratio $r_g = g_{m1}/g_{m2} \approx 1$ is smaller than 0.5%. For a larger transconductance ratio $r_g \gg 1$, the tolerance can be 2%.

Another limitation of OTAs can be a small input voltage swing required to maintain linearity.

All of these technological imperfections must be taken into consideration in filter design.

The parasitic effects and the large parameter tolerances are the disadvantages of OTA filters. Therefore, it is necessary to include an on-chip tuning system to compensate for the technological imperfections. We can achieve a better performance of OTA filters by adjusting the transfer function parameters, so that the transfer-function pole magnitude can be successfully tuned by the on-chip tuning system. The OTA filter is often implemented on the same chip with the self-tuning system.

OTA filters can be designed using the doubly terminated *RLC* realization or as a cascade connection of biquads. It has been shown that the dynamic range of the ladder filters is superior to the dynamic range of the cascaded biquad filters. Also, the ladder filters are less sensitive to the tolerances than the cascaded biquads. The main disadvantage of a higher-order selective ladder filter is its complex and less precise tuning procedure. For high-frequency applications and sharp specifications, realizations with cascaded biquads are more attractive because they can be precisely tuned.

4.11.1 Biquadratic OTA-C Realizations

In this chapter we present filter realizations with OTAs and capacitors, only. We consider several types of OTA biquads, including very simple realizations with the minimal number of elements (OTAs and capacitors).

Simple OTA-C Biquads. A simple universal OTA biquad realized with only four elements is shown in Fig. 4.66. This biquad can be used as a lowpass, highpass, bandpass, or bandreject second-order filter section. The required transfer function is realized by connecting the terminals V_a, V_b, and V_c to the ground or to the input voltage source V_g. The output voltage is V_3.

The realizable transfer functions of the biquad from Fig. 4.66 are

$$\frac{V_3}{V_g} = \frac{s^2 \dfrac{V_c}{V_g} + s\dfrac{g_{m2}}{C_2}\dfrac{V_b}{V_g} + \dfrac{g_{m1}g_{m2}}{C_1C_2}\dfrac{V_a}{V_g}}{s^2 + \dfrac{g_{m2}}{C_2}s + \dfrac{g_{m1}g_{m2}}{C_1C_2}}$$

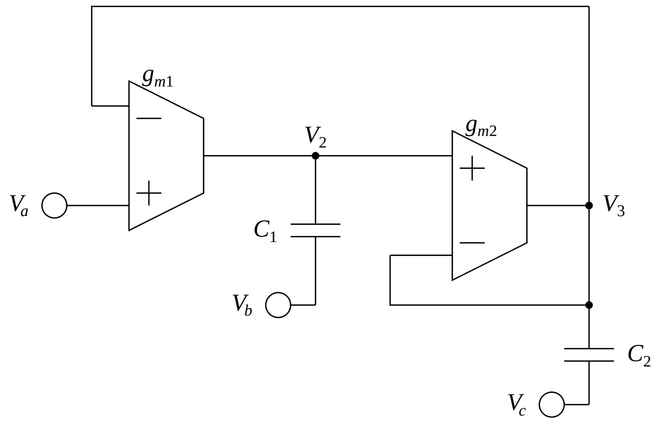

Figure 4.66 Four-element universal OTA-C biquad.

$$
\begin{aligned}
H_{LP}(s) &= \frac{\frac{g_{m1}g_{m2}}{C_1C_2}}{s^2 + \frac{g_{m2}}{C_2}s + \frac{g_{m1}g_{m2}}{C_1C_2}}, \quad V_a = V_g,\ V_b = 0,\ V_c = 0 \\
H_{HP}(s) &= \frac{s^2}{s^2 + \frac{g_{m2}}{C_2}s + \frac{g_{m1}g_{m2}}{C_1C_2}}, \quad V_a = 0,\ V_b = 0,\ V_c = V_g \\
H_{BP}(s) &= \frac{s\frac{g_{m2}}{C_2}}{s^2 + \frac{g_{m2}}{C_2}s + \frac{g_{m1}g_{m2}}{C_1C_2}}, \quad V_a = 0,\ V_b = V_g,\ V_c = 0 \\
H_{BR}(s) &= \frac{s^2 + \frac{g_{m1}g_{m2}}{C_1C_2}}{s^2 + \frac{g_{m2}}{C_2}s + \frac{g_{m1}g_{m2}}{C_1C_2}}, \quad V_a = V_g,\ V_b = 0,\ V_c = V_g
\end{aligned}
\tag{4.190}
$$

The pole magnitude, ω_p, and the pole Q-factor, Q_p, are given by

$$
\begin{aligned}
\omega_p &= \sqrt{\frac{g_{m1}g_{m2}}{C_1C_2}} \\
Q_p &= \sqrt{\frac{g_{m1}C_2}{g_{m2}C_1}}
\end{aligned}
\tag{4.191}
$$

The Q-factor is determined by the capacitance ratio, $\frac{C_2}{C_1}$, and the transconductance ratio, $\frac{g_{m1}}{g_{m2}}$, which can be accurately maintained in monolithic design. The most sensitive parameter, ω_p, is a function of the transconductance-capacitance ratio, $\frac{g_m}{C}$, which is difficult to manufacture accurately.

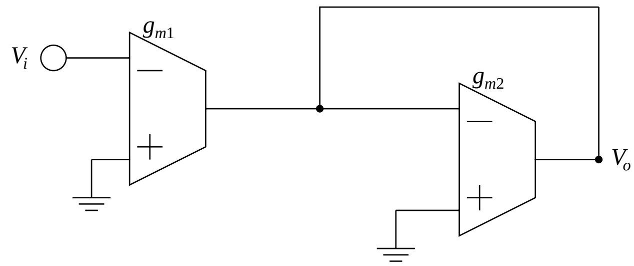

Figure 4.67 Inverting all-OTA amplifier.

For higher-order filters obtained by cascading these biquads, a buffer (voltage follower) is required to prevent interstage loading. Only lowpass filter sections can be cascaded without using buffers because the output V_3 of one biquad can be directly connected to the input V_a of the next biquad.

Typical inverting and noninverting all-OTA amplifiers are shown in Figs. 4.67 and 4.68. The gain of the inverting amplifier is

$$\frac{V_o}{V_i} = -\frac{g_{m1}}{g_{m2}}$$

and the gain of the noninverting amplifier is

$$\frac{V_o}{V_i} = \frac{g_{m1}}{g_{m2}}$$

For $g_{m1} = g_{m2}$ the gain is -1 or 1.

Simple OTA-C Lowpass Notch Biquad. The universal biquad from Fig. 4.66 can be modified to become a second-order lowpass notch filter section as shown in Fig. 4.69. The capacitor C_2 in Fig. 4.66 has been split into two capacitors C_2 and C_3. The transfer function is found to be

$$H(s) = \frac{V_3}{V_1} = \frac{C_2}{C_2 + C_3} \frac{s^2 + \dfrac{g_{m1}g_{m2}}{C_1C_2}}{s^2 + \dfrac{g_{m2}}{C_2 + C_3}s + \dfrac{g_{m1}g_{m2}}{C_1(C_2 + C_3)}} \tag{4.192}$$

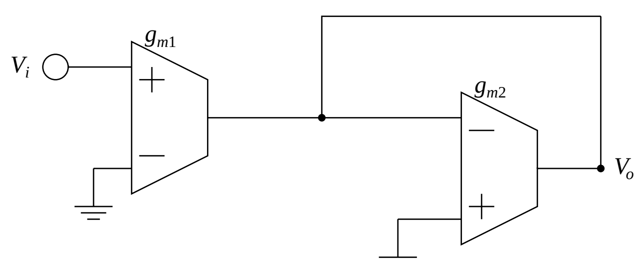

Figure 4.68 Noninverting all-OTA amplifier.

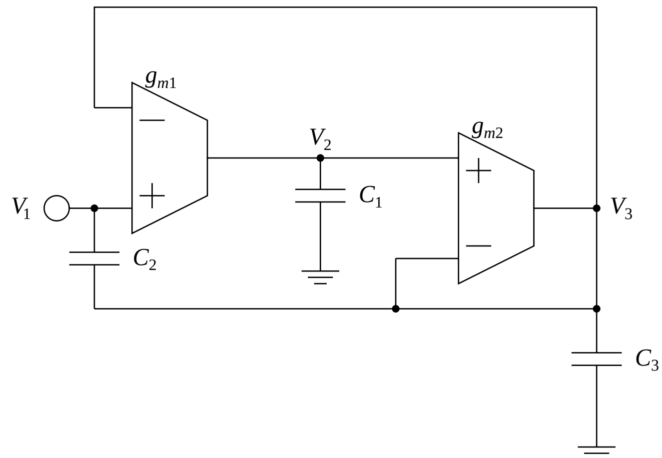

Figure 4.69 Simple lowpass notch OTA-C biquad.

The pole magnitude, ω_p, the pole Q-factor, Q_p, and the magnitude of the transfer function zero, ω_z, are

$$\omega_p = \sqrt{\frac{g_{m1}g_{m2}}{C_1(C_2 + C_3)}}$$
$$Q_p = \sqrt{\frac{g_{m1}(C_2 + C_3)}{g_{m2}C_1}} \qquad (4.193)$$
$$\omega_z = \sqrt{\frac{g_{m1}g_{m2}}{C_1C_2}}$$

Obviously,

$$\frac{\omega_p}{\omega_z} = \sqrt{\frac{C_2}{C_2 + C_3}} < 1 \qquad (4.194)$$

which implies that we can realize only the lowpass notch transfer function with this biquad.

Notice that the gain constant $\frac{C_2}{C_2+C_3}$ is always smaller than 1.

The disadvantages of this realization are as follows: (a) It requires three capacitors instead of two, and (b) buffers between biquads are required in cascade realizations of higher-order filters. Therefore, at least seven components (four OTAs and three capacitors) are required for a lowpass notch biquad.

General OTA-C Biquads. A universal OTA-C second-order filter section that does not require any additional buffers is shown in Fig. 4.70.

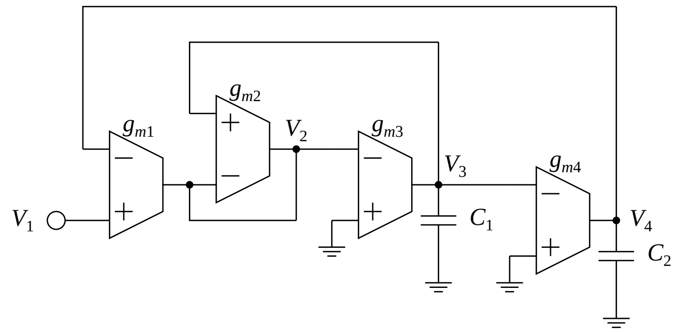

Figure 4.70 Four-OTA general biquad.

This biquad can realize the lowpass $H_{LP}(s)$, highpass $H_{HP}(s)$, and bandpass $H_{BP}(s)$ transfer function:

$$
\begin{aligned}
H_{LP}(s) &= \frac{V_4}{V_1} = \frac{\omega_p^2}{s^2 + \dfrac{\omega_p}{Q_p}s + \omega_p^2} \\
H_{BP}(s) &= \frac{V_3}{V_1} = \frac{\dfrac{\omega_p}{Q_p}s}{s^2 + \dfrac{\omega_p}{Q_p}s + \omega_p^2} \\
H_{HP}(s) &= \frac{V_2}{V_1} = \frac{s^2}{s^2 + \dfrac{\omega_p}{Q_p}s + \omega_p^2} \\
\omega_p &= \sqrt{\frac{g_{m1}g_{m3}}{C_1C_2}\frac{g_{m4}}{g_{m2}}} \\
Q_p &= \sqrt{\frac{C_1g_{m1}}{C_2g_{m3}}\frac{g_{m4}}{g_{m2}}}
\end{aligned}
\tag{4.195}
$$

The design equations for this biquad are simple: We can assign arbitrary values to g_{m1}, g_{m2}, C_1, and C_2, and then we can calculate the other transconductances g_{m3} and g_{m4} from

$$
\begin{aligned}
g_{m3} &= \frac{\omega_p}{Q_p}C_1 \\
g_{m4} &= \omega_p Q_p C_2 \frac{g_{m2}}{g_{m1}}
\end{aligned}
\tag{4.196}
$$

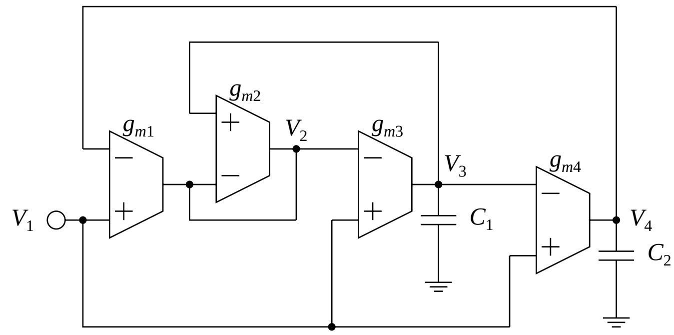

Figure 4.71 Four-OTA notch biquad.

For $g_{m1} = g_{m2}$ and $C_1 = C_2 = C$, the Q-factor can be tuned by the ratio $\frac{g_{m4}}{g_{m3}}$, and the pole magnitude can be tuned by the product $g_{m3}g_{m4}$:

$$\omega_p = \sqrt{\frac{g_{m3}g_{m4}}{C^2}}$$
$$Q_p = \sqrt{\frac{g_{m4}}{g_{m3}}}$$

which means that we have to tune only two biquad parameters, g_{m3} and g_{m4}.

A modification of the realization shown in Fig. 4.70, performed by connecting the + inputs of the third and fourth OTA to the input V_1, is presented in Fig. 4.71. Such a realization can be used as a second-order notch filter section.

Another versatile OTA-C biquad is shown in Fig. 4.72.

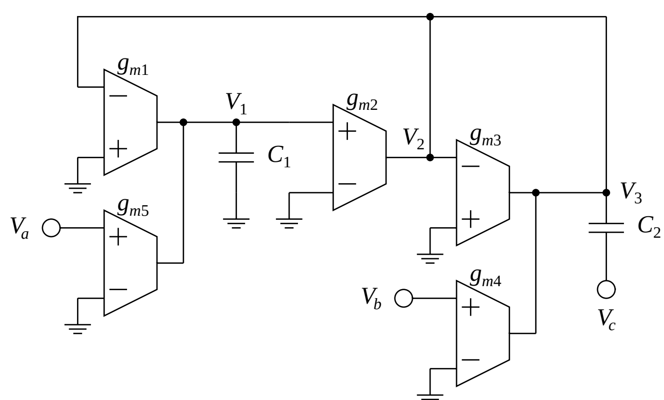

Figure 4.72 Five-OTA universal biquad.

The realizable transfer-function types are lowpass, highpass, and bandpass:

$$
\begin{aligned}
\frac{V_3}{V_g} &= \frac{s^2 \dfrac{V_c}{V_g} + s\dfrac{g_{m4}}{C_2}\dfrac{V_b}{V_g} + \dfrac{g_{m2}g_{m5}}{C_1C_2}\dfrac{V_a}{V_g}}{s^2 + \dfrac{g_{m3}}{C_2}s + \dfrac{g_{m1}g_{m2}}{C_1C_2}} \\
H_{LP}(s) &= \frac{\dfrac{g_{m2}g_{m5}}{C_1C_2}}{s^2 + \dfrac{g_{m3}}{C_2}s + \dfrac{g_{m1}g_{m2}}{C_1C_2}}, \quad V_a = V_g, \quad V_b = 0, \quad V_c = 0 \\
H_{HP}(s) &= \frac{s^2}{s^2 + \dfrac{g_{m3}}{C_2}s + \dfrac{g_{m1}g_{m2}}{C_1C_2}}, \quad V_a = 0, \quad V_b = 0, \quad V_c = V_g \\
H_{BP}(s) &= \frac{s\dfrac{g_{m4}}{C_2}}{s^2 + \dfrac{g_{m3}}{C_2}s + \dfrac{g_{m1}g_{m2}}{C_1C_2}}, \quad V_a = 0, \quad V_b = V_g, \quad V_c = 0
\end{aligned}
\tag{4.197}
$$

The bandreject, lowpass-notch, and highpass-notch transfer functions can be realized for $V_b = 0$ and $V_a = V_c = V_g$, and by proper selection of the ratio of transconductances g_{m1} and g_{m5} we obtain

$$
\begin{aligned}
H_{BR}(s) &= \frac{s^2 + \dfrac{g_{m2}g_{m5}}{C_1C_2}}{s^2 + \dfrac{g_{m3}}{C_2}s + \dfrac{g_{m1}g_{m2}}{C_1C_2}}, \quad g_{m1} = g_{m5} \\
H_{LPN}(s) &= \frac{s^2 + \dfrac{g_{m2}g_{m5}}{C_1C_2}}{s^2 + \dfrac{g_{m3}}{C_2}s + \dfrac{g_{m1}g_{m2}}{C_1C_2}}, \quad g_{m1} < g_{m5} \\
H_{HPN}(s) &= \frac{s^2 + \dfrac{g_{m2}g_{m5}}{C_1C_2}}{s^2 + \dfrac{g_{m3}}{C_2}s + \dfrac{g_{m1}g_{m2}}{C_1C_2}}, \quad g_{m1} > g_{m5}
\end{aligned}
\tag{4.198}
$$

The design equations can be readily derived from the above transfer functions.

4.12 CURRENT-CONVEYOR (CC) FILTERS

In this section we consider design of continuous-time active filters using current conveyors (CC).

The actual trend in integrated-circuits technology is to incorporate mixed analog and digital functions on a single chip. In order to use very small silicon area and to manufacture low cost mixed-mode integrated circuits, it is preferable to use the same technologies for digital and analog circuits. Generally, the technology has been optimized for digital circuits. Therefore, the mixed-mode (analog and digital) integrated circuits require an advanced level of analog continuous-time filter design.

Analog continuous-time active filters consist of active amplifiers and passive components like capacitors and resistors. The basic element of an amplifier is a transistor. The small-signal model of a transistor is a voltage-controlled current output device. Usually, we assemble transistors into voltage-oriented circuits because we tend to think in voltage terms rather than in current terms. This "voltage" approach reduces the useful frequency range 10 or 100 times the useful frequency range of the "current" approach.

In this book we consider only the second generation of current conveyors, denoted as CCII, which is more useful than the first generation, CCI, for filter design.

A great deal of work has been reported on the design of CC filters. The frequency range of the CC filters can be up to GHz. The useful frequency range of current conveyors is limited only by the transistor gain–bandwidth product.

The most commonly used circuit symbol for CCII is shown in Fig. 4.73. The ideal CCII is described by three equations:

$$\begin{aligned} I_y &= 0 \\ V_x &= V_y \\ I_z &= aI_x \end{aligned} \tag{4.199}$$

The input terminal Y exhibits an infinite input impedance, and the current that flows into that terminal is identically zero. The voltage applied to terminal Y appears at terminal X. The current flowing into terminal X is conveyed to the output terminal Z. The real constant a is the *current gain* of CCII. Traditionally, the CCII operation has largely been associated with the unity current gain. For $a = 1$ the device is called the positive current conveyor and denoted by CCII+. For $a = -1$ the device is called the negative current conveyor and denoted by CCII–. However, a nonunity current gain is just as easily implemented [48, p. 115].

Most CC filters are based on the following:

- Simulating inductors of a passive doubly terminated RLC filter with CCs, capacitors, and resistors
- Converting an op amp active RC filter into a CC active RC equivalent filter

The inductor simulation is attractive because the corresponding CC active filter retains low passive-component sensitivities. The conversion of op amp active RC circuits into CC active RC circuits retains exactly the same component sensitivities, and we can use

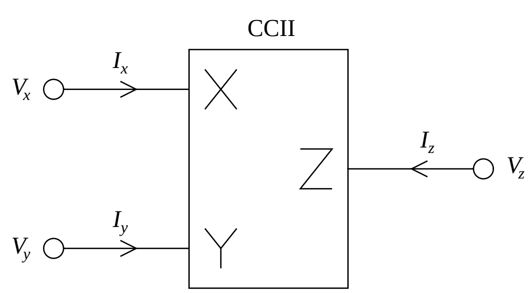

Figure 4.73 The second generation current-conveyor (CCII) circuit symbol.

the well-known theory of op amp active RC filters without having to reinvent the new low-sensitive CC circuits.

4.12.1 Current-Conveyor Filters Based on Passive RLC Filters

The current conveyor is an active device that is ideally suited for integrated-circuits technology. Thus, the filter design with CCs should rely on inductorless realizations because inductors cannot be successfully integrated. A straightforward way to obtain a CC filter is to start from a passive RLC realization and simulate the inductors by CCs, capacitors and resistors.

A grounded inductor can be simulated with the one-port active RC network shown in Fig. 4.74a. The input impedance of the one-port is

$$Z_{in1} = \frac{V_1}{I_1} = \frac{CR_1R_2}{-\,a_1a_2}s \tag{4.200}$$

The input impedance of the one-port network containing single inductor (Fig. 4.74b), is

$$Z_{in2} = \frac{V_2}{I_2} = Ls \tag{4.201}$$

To make the two one-ports equivalent, $Z_{in1} = Z_{in2}$ must hold, or

$$L = \frac{CR_1R_2}{-\,a_1a_2} > 0 \tag{4.202}$$

We conclude that one CC must be CCII+, and the other must be CCII–. The simplest case is the unity gain conveyor, $a_1 = 1$ and $a_2 = -1$, which yields $L = CR_1R_2$.

Consider a doubly terminated passive RLC realization of the fifth-order filter displayed in Fig. 4.75. If we replace the grounded inductors with the one-port active

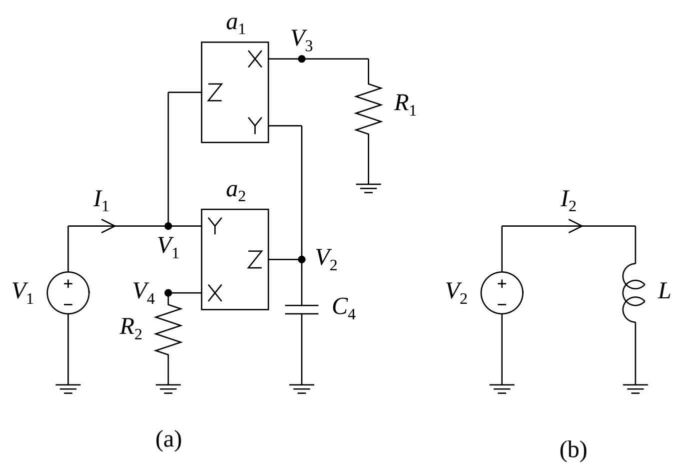

Figure 4.74 (a) Current conveyor circuit that simulates a grounded inductor (b) Simple circuit with the grounded inductor.

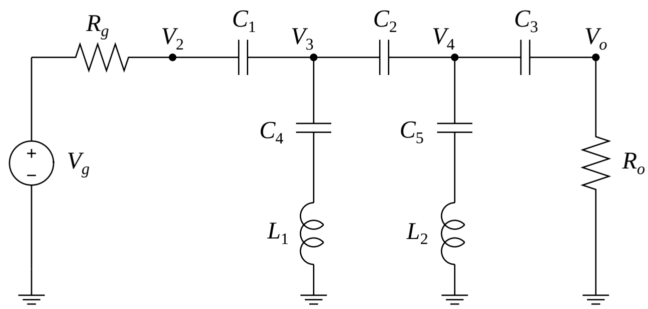

Figure 4.75 Fifth-order highpass filter.

RC network from Fig. 4.74, we obtain a CC active *RC* realization. The transfer function of the CC realization is

$$
\begin{aligned}
H(s) &= H_1(s)H_2(s)\\
H_1(s) &= \frac{1+C_4Ls^2}{C_1+C_4+C_1C_4Rs+C_1C_4Ls^2}\\
H_2(s) &= \frac{C_1^2C_2R(1+C_4Ls^2)s}{C_1+2C_2+C_4+2C_1C_2Rs+C_1C_4Rs+C_1C_4Ls^2+2C_2C_4Ls^2+2C_1C_2C_4LRs^3}\\
R_g &= R_o = R, \qquad C_3 = C_1, \qquad C_5 = C_4, \qquad L_2 = L_1 = L
\end{aligned}
$$

It is important to notice that the CC filters derived from doubly terminated ladder realizations inherit low sensitivities to passive components.

4.12.2 Current-Conveyor Filters Derived from Op Amp Active *RC* Filters

Well-established theory and practice of monolithic op amp active *RC* filters can serve as a basis for obtaining high performance current-conveyor filters. In this section we demonstrate how op amp active *RC* realizations can be directly converted to CC realizations with the same transfer function and component sensitivities.

The conversion is based on the reciprocal behavior of two-port networks. *Two-port reciprocity* means that a voltage, say V, applied across the input port of a linear two-port network produces a short-circuited output current that is identical to the short-circuited input port current that would result if V were to be removed from the input port and impressed instead across the output port [16, p. 544].

In other words, a two-port network is considered reciprocal when the same transfer function results as the excitation and the response are interchanged (Fig. 4.76a):

$$\frac{V_o}{V_g} = \frac{I_o}{I_g} \tag{4.203}$$

where V_g is the voltage of the voltage source, I_g is the current of the current source, V_o is the voltage of the open-circuited port, and I_o is the current through the short-circuited port.

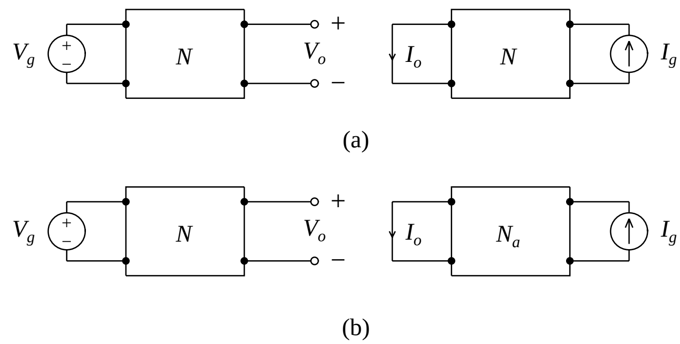

Figure 4.76 Definition of (a) reciprocal network and (b) interreciprocal networks, $\frac{V_o}{V_g} = \frac{I_o}{I_g}$.

Two different two-port networks (Fig. 4.76b), satisfying the Eq. (4.203) are called *interreciprocal networks*. An interreciprocal network N_a to a given network N is called an *adjoint network*. The adjoint network is obtained from the network N by using the following conversion rules (Fig. 4.77):

- Passive R and C elements in N_a are the same as those in N.
- The ideal input voltage source of N is converted to a short circuit, and the current through it becomes the output of N_a.
- The port of N at which the output voltage is taken becomes a current source in N_a.

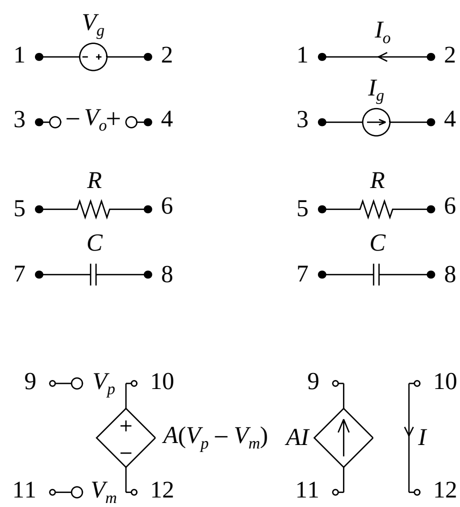

Figure 4.77 Conversion rules for deriving adjoint network.

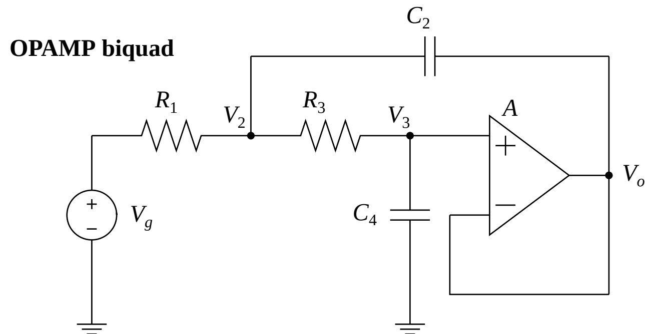

Figure 4.78 Sallen–Key lowpass op amp biquad.

- A voltage-controlled voltage source is converted to a current-controlled current source: The controlled voltage source, AV, is converted to a short circuit through which flows the controlling current I, while the controlling voltage V is converted to a controlled current source, AI.

For example, the Sallen–Key lowpass low-Q-factor op amp realization (Fig. 4.78) can be directly converted to a CC realization shown in Fig. 4.79. The current conveyor with the grounded Y terminal acts as a current amplifier. The transfer function of both circuits is the lowpass transfer function:

$$H(s) = \frac{\dfrac{1}{C_2C_4R_1R_3}}{s^2 + \dfrac{R_1 + R_3}{C_2R_1R_3}s + \dfrac{1}{C_2C_4R_1R_3}}$$

A fourth-order highpass filter realized as a cascade of two biquads is converted to the corresponding current-conveyor filter as shown in Figs. 4.80 and 4.81.

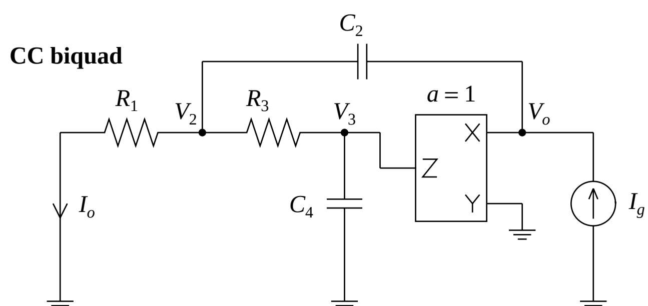

Figure 4.79 Sallen–Key lowpass current-conveyor biquad.

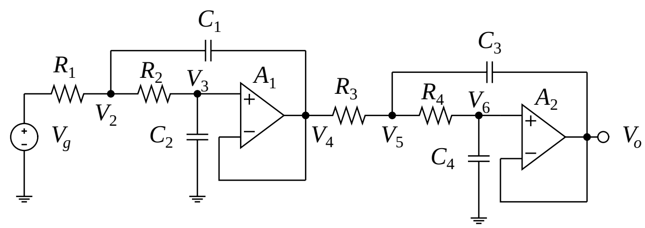

Figure 4.80 Cascaded op amp biquads.

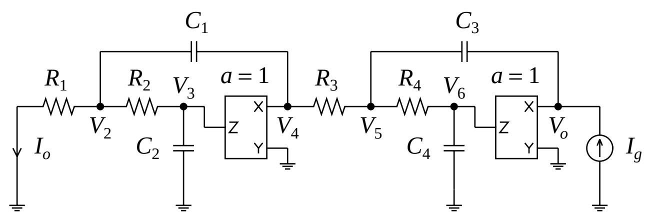

Figure 4.81 Cascaded current-conveyor biquads.

■ PROBLEMS

4.1 Find the transfer function coefficients of the second-order lowpass ($c = 1$, $a = b = 0$), highpass ($a = 1$, $b = c = 0$), and bandpass ($b = Q_p$, $a = c = 0$) filter, and find the maximum of the magnitude response if it occurs at $f_e = 10$ kHz

$$H(s) = \frac{as^2 + b\dfrac{\omega_p}{Q_p}s + c\omega_p^2}{s^2 + \dfrac{\omega_p}{Q_p}s + \omega_p^2}$$

Assume $Q_p = 2$.

4.2 Sketch the magnitude response of the second-order lowpass, highpass, and bandpass filter with the same $Q_p = 100$ and the maximum occurring at $f_e = 10$ kHz. What do you conclude about relationship between the Q-factor and the magnitude response?

4.3 Sketch the magnitude response of the second-order lowpass, highpass, and bandreject ($a = c = Q_p$, $b = 0$) filter with the same $Q_p = 10$ and $\omega_p = 10^4$ rad/s. Determine the 3-dB frequencies.

4.4 Sketch the magnitude response of the second-order filter

$$H(s) = \frac{s^2 + \dfrac{\omega_z}{Q_z}s + \omega_z^2}{s^2 + \dfrac{\omega_p}{Q_p}s + \omega_p^2}$$

for $Q_p = 2$, $Q_z = 1000$, and $\omega_p = 10$ rad/s and with

(a) $\omega_z = 3$ rad/s,

(b) $\omega_z = 6$ rad/s,

(c) $\omega_z = 9$ rad/s,
(d) $\omega_z = 10$ rad/s,
(e) $\omega_z = 10.5$ rad/s,
(f) $\omega_z = 12$ rad/s,
(g) $\omega_z = 15$ rad/s.

4.5 Sketch the magnitude response of the second-order filter

$$H(s) = \frac{s^2 + \dfrac{\omega_z}{Q_z}s + \omega_z^2}{s^2 + \dfrac{\omega_p}{Q_p}s + \omega_p^2}$$

for $Q_p = 3$, $\omega_z = 11$ rad/s, and $\omega_p = 10$ rad/s and with

(a) $Q_z = 1$,
(b) $Q_z = 3$,
(c) $Q_z = 10$,
(d) $Q_z = 100$,
(e) $Q_z = 1000$.

4.6 Decompose the fourth-order transfer function

$$H_4(s) = \frac{\left(\dfrac{\omega_p}{Q_p}s\right)^2}{\left(s^2 + \dfrac{\omega_p}{Q_p}s + \omega_p^2\right)^2}$$

into second-order transfer functions:

(a) into a product of a lowpass and a highpass transfer function and
(b) into a product of two bandpass transfer functions.

For $Q_p = 10$ and $\omega_p = 10$ rad/s, find the maximal values of the magnitude responses of each second-order transfer function. Suggest a decomposition suitable for implementation and give reasons for your choice.

4.7 Decompose the second-order transfer function

$$H_2(s) = \frac{\omega_p^2}{s^2 + \dfrac{\omega_p}{0.5 + q_p}s + \omega_p^2}$$

into a product of two first-order transfer functions for $\omega_p = 1$ rad/s and $q_p = -0.25$. Can you find q_p that yields the minimum pole Q-factor of $H_2(s)$.

4.8 Consider the lowpass low-Q-factor op amp biquad. Find the transfer function parameters K, Q_p, and ω_p

$$H(s) = K\frac{\omega_p^2}{s^2 + \dfrac{\omega_p}{Q_p}s + \omega_p^2}$$

in terms of element values. Assume $R_{11} = 200\ \text{k}\Omega$, $R_{12} = 100\ \text{k}\Omega$, $R_3 = 50\ \text{k}\Omega$, $C_2 = 100$ nF, $C_4 = 10$ nF. Compute the passive sensitivities of ω_p, K, and Q_p.

4.9 Consider the biquad from Problem 4.8. Assume that a parasitic capacitor, $C = 100$ pF, exists between each node and ground. Derive the transfer function and calculate the magnitude response deviation in dB with respect to the ideal case ($C = 0$).

4.10 Consider the biquad from Problem 4.8 which is driven by a voltage source with the serial internal resistance $R_s = 10\ \mathrm{k}\Omega$. Find the transfer function and calculate the magnitude response deviation in dB with respect to the ideal case ($R_s = 0$).

4.11 Consider the biquad from Problem 4.8 with finite and frequency dependent op amp gain A that can be approximated by

$$A = 10^5 \frac{2\pi 10^2}{s + 2\pi 10^2} \frac{2\pi 10^4}{s + 2\pi 10^4} \frac{\pi 10^6}{s + \pi 10^6}$$

Find the transfer function and calculate the magnitude response deviation in dB with respect to the ideal case ($A \to +\infty$).

4.12 Consider the biquad from Problem 4.8 with the resistor and capacitor tolerances $\pm 2\%$, the resistor temperature coefficient 1000 ppm/K, and the capacitor temperature coefficient -150 ppm/K. The filter is required to work over the temperature range from $T_1 = 250$ K to $T_2 = 350$ K. Calculate the maximum magnitude response deviation in dB with respect to the ideal case (zero tolerances and temperature independent components). Assume the room temperature $T_0 = 293$ K.

MATLAB EXERCISES

4.1 Write a MATLAB script to plot the magnitude responses of the second-order lowpass, highpass, and bandpass transfer functions given in Problem 4.1 for $0 \leq f \leq 30$ kHz. Verify your results with the function `freqs`.

4.2 Write a MATLAB script to plot the magnitude responses of the second-order lowpass, highpass, and bandpass transfer functions given in Problem 4.2 for $0 \leq f \leq 30$ kHz. Verify your results with the function `freqs`.

4.3 Write a MATLAB script to plot the magnitude responses of the second-order transfer functions given in Problem 4.3 for $0 \leq f \leq 3$ kHz. Verify your results with the function `freqs`.

4.4 Write a MATLAB script to plot the magnitude responses of the second-order transfer functions given in Problem 4.4 for $0 \leq \omega \leq 20$ rad/s. Which ones of the transfer functions are lowpass-notch transfer functions?

4.5 Write a MATLAB script to plot the magnitude responses of the second-order transfer functions given in Problem 4.5 for $0 \leq \omega \leq 20$ rad/s. Which ones of the transfer functions are lowpass-notch transfer functions?

4.6 Write a MATLAB script to plot the magnitude responses of the transfer functions given in Problem 4.6 for $0 \leq \omega \leq 20$ rad/s. Which one of the transfer functions has the largest maximum of the magnitude responses?

4.7 Write a MATLAB script to plot the magnitude response of the second-order transfer function given in Problem 4.7 for $0 \leq f \leq 1$ Hz. Plot the transfer function poles for $q_p \in \{-1, -0.5, 0, 0.5, 1, 1.5, 2\}$. In which case is the transfer function causal?

4.8 Write a MATLAB script to plot the magnitude response of the transfer function given in Problem 4.8. Write a MATLAB program to calculate K, Q_p, and ω_p in terms of R_{11}, R_{12}, R_3, C_2, and C_4. If C_2 changes by $+10\%$, while C_4 changes by -10%, calculate the magnitude response deviation in dB.

4.9 Write MATLAB programs to (1) draw the circuit schematic, (2) compute the transfer function, poles, zeros, and Q-factors in terms of element values, (3) find the element values in terms of design parameters, (4) verify the filter realization, and plot the frequency response of the following second-order filters:

- **(a)** lowpass low-Q-factor op amp active RC second-order filter,
- **(b)** highpass low-Q-factor op amp active RC filter
- **(c)** bandpass low-Q-factor op amp active RC filter
- **(d)** bandreject low-Q-factor op amp active RC filter
- **(e)** allpass low-Q-factor op amp active RC filter
- **(f)** lowpass medium-Q-factor op amp active RC filter
- **(g)** highpass medium-Q-factor op amp active RC filter
- **(h)** bandpass medium-Q-factor op amp active RC filter
- **(i)** bandreject medium-Q-factor op amp active RC filter
- **(j)** allpass medium-Q-factor op amp active RC filter
- **(k)** lowpass notch medium-Q-factor op amp active RC filter
- **(l)** highpass notch medium-Q-factor op amp active RC filter
- **(m)** lowpass high-Q-factor op amp active RC filter
- **(n)** highpass high-Q-factor op amp active RC filter
- **(o)** bandpass high-Q-factor op amp active RC filter
- **(p)** bandreject high-Q-factor op amp active RC filter
- **(q)** allpass high-Q-factor op amp active RC filter
- **(r)** lowpass-notch high-Q-factor op amp active RC filter
- **(s)** highpass-notch high-Q-factor op amp active RC filter
- **(t)** general-purpose op amp active RC filter
- **(u)** OTA-C general biquad

■ *MATHEMATICA* EXERCISES

4.1 Write a *Mathematica* code to plot the magnitude responses of the second-order lowpass, highpass, and bandpass transfer functions given in Problem 4.1 for $0 \le f \le 30$ kHz. Determine the 3-dB frequencies.

4.2 Write a *Mathematica* code to plot the magnitude responses of the second-order lowpass, highpass, and bandpass transfer functions given in Problem 4.2 for $0 \le f \le 30$ kHz. Determine the 3-dB frequencies.

4.3 Write a *Mathematica* code to plot the magnitude responses of the second-order transfer functions given in Problem 4.3 for $0 \le f \le 3$ kHz.

4.4 Write a *Mathematica* code to plot the magnitude responses of the second-order transfer functions given in Problem 4.4 for $0 \le \omega \le 20$ rad/s. Which ones of the transfer functions are lowpass-notch transfer functions?

4.5 Write a *Mathematica* code to plot the magnitude responses of the second-order transfer functions given in Problem 4.5 for $0 \le \omega \le 20$ rad/s. Which ones of the transfer functions are lowpass-notch transfer functions?

4.6 Write a *Mathematica* code to plot the magnitude responses of the transfer functions given in Problem 4.6 for $0 \le \omega \le 20$ rad/s. Which one of the transfer function has the largest maximum of the magnitude responses?

4.7 Write a *Mathematica* code to plot the magnitude response of the second-order transfer functions given in Problem 4.7 for $0 \le f \le 1$ Hz. Plot the poles for $q_p \in \{-1, -0.5, 0, 0.5, 1, 1.5, 2\}$. In which case is the transfer function causal?

4.8 Write a *Mathematica* code to plot the magnitude response of the transfer function given in Problem 4.8. Write a *Mathematica* code to calculate K, Q_p, and ω_p in terms of R_{11}, R_{12}, R_3, C_2, and C_4. If C_2 changes by $+10\%$ while C_4 changes by -10%, calculate the magnitude response deviation in dB.

4.9 Write *Mathematica* programs to (1) draw the circuit schematic and formulate circuit equations from the schematic, (2) compute the transfer function, poles, zeros, and Q-factors in terms of element values, (3) find sensitivity functions and the gain-sensitivity product, (4) find the element values in terms of design parameters, (5) verify the filter realization, and plot the frequency response of the following second-order filters:

(a) lowpass low-Q-factor op amp active RC second-order filter,
(b) highpass low-Q-factor op amp active RC filter
(c) bandpass low-Q-factor op amp active RC filter
(d) bandreject low-Q-factor op amp active RC filter
(e) allpass low-Q-factor op amp active RC filter
(f) lowpass medium-Q-factor op amp active RC filter
(g) highpass medium-Q-factor op amp active RC filter
(h) bandpass medium-Q-factor op amp active RC filter
(i) bandreject medium-Q-factor op amp active RC filter
(j) allpass medium-Q-factor op amp active RC filter
(k) lowpass notch medium-Q-factor op amp active RC filter
(l) highpass notch medium-Q-factor op amp active RC filter
(m) lowpass high-Q-factor op amp active RC filter
(n) highpass high-Q-factor op amp active RC filter
(o) bandpass high-Q-factor op amp active RC filter
(p) bandreject high-Q-factor op amp active RC filter
(q) allpass high-Q-factor op amp active RC filter
(r) lowpass-notch high-Q-factor op amp active RC filter
(s) highpass-notch high-Q-factor op amp active RC filter
(t) general-purpose op amp active RC filter
(u) OTA-C general biquad